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Connections of Fiber-Reinforced Polymer (FRP) Structural Members: A Review of the State of the Art

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ABSTRACT

This review of the state of the art of connections of fiber-reinforced polymer (FRP) members covers primary structural connections (whose integrity is crucial to the functioning and safety of a structure), in which at least one of the members is made of FRP. The connections consist of mechanical joints, bonded joints, interlocking joints, or a combination of those, such as frame connections. After an earlier period of benefiting from aerospace research and imitation of steel connections, FRP connection design in civil infrastructure is beginning to develop in new, original directions

Keywords: adhesive, bolt, bond, composite, connection, fiber-reinforced polymer, frame, FRP, interlocking, joint.

ACKNOWLEDGMENT

DISCLAIMER

Trade or manufacturer's names appear herein because they are essential to the objectives of this document. The United States government does not endorse products or manufacturers.

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Chapter 1
Introduction

Fiber-reinforced polymers (FRP) represent a new class of construction materials. In the past two decades, their use has spread from the aerospace industry to civil infrastructure, which presents a new set of challenges. For seismic rehabilitation, such as wrapping of bridge columns, and applications where corrosion is an important consideration, FRP are competitive with more traditional construction materials. Compared to these, FRP offer significant advantages, such as resistance to corrosion, high strength, light weight and ease of installation. Their disadvantages include high initial cost, possible durability problems in the presence of moisture, ultra-violet radiation, freeze-thaw cycles, or sustained stresses, and the absence of design standards and experience.

FRP can be made into structural shapes by pultrusion, a process that combines extrusion and pulling of molten or curable resin and continuous fibers usually arranged in unidirectional layers, or plies, through a die of a desired structural shape of constant cross-section. The assembly of structural members, made of FRP or other materials, requires connections, which are therefore integral parts of a structure and can be classified as:

- Primary structural connections, whose integrity is crucial to the function and safety of a structure. They are designed to provide major strength and stiffness to the structural assembly for the life of the structure.
- Secondary structural connections, which provide some strength and stiffness to the assembly, but their failure would not cause damage beyond the structural components they join. An example is the field connection of two modular units.
- Non-structural connections, e.g., joints of decorative panels.

Although FRP structural members can be made in a variety of shapes (Fig. 1.1), large volume construction requires standard shapes, which must be limited to certain sizes for ease of transportation and assembly. In addition, individual building details vary greatly in geometry, making assembly of standardized components much more practical than custom fabricated members. For all these reasons, connections are necessary components of a structure. Figures 1.2, 1.3 and 1.4 show examples of FRP connections in civil structures. It is apparent that they are similar to steel bolted connections, an expected development in this early stage of use of a relatively new material such as FRP. It will be seen in this review that there are, in fact, important differences between the behavior and design of steel and FRP connections.

This review of the state of the art of connections of FRP members covers primary structural connections, in which at least one of the members is made of FRP. The connections consist of mechanical joints (Chapter 2), bonded joints (Chapter 3), interlocking joints (Chapter 5), or a combination of those, such as frame connections (Chapter 4). For more details, the reader is referred to excellent reviews of the subject, such as Hutchinson's (1997) "Joining of Fibre-Reinforced Polymer Composite Structures", and Matthews's (1987) "Joining Fibre-Reinforced Plastics". Also of interest is the EUROCOMP Design Guide and Handbook (1996), "Structural Design of Polymer Composites", which is a comprehensive design guide for FRP structures following a Load and Resistance Factor Design (LRFD) format.

1.1   References

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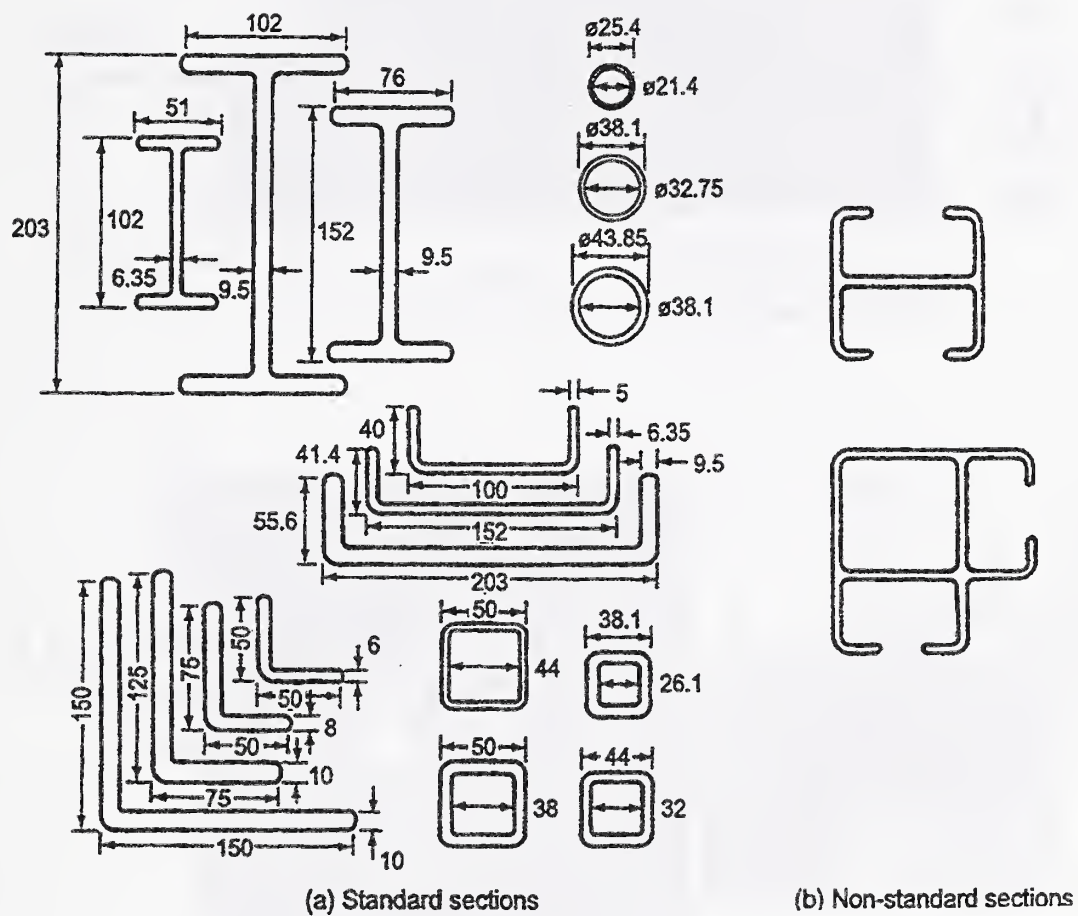
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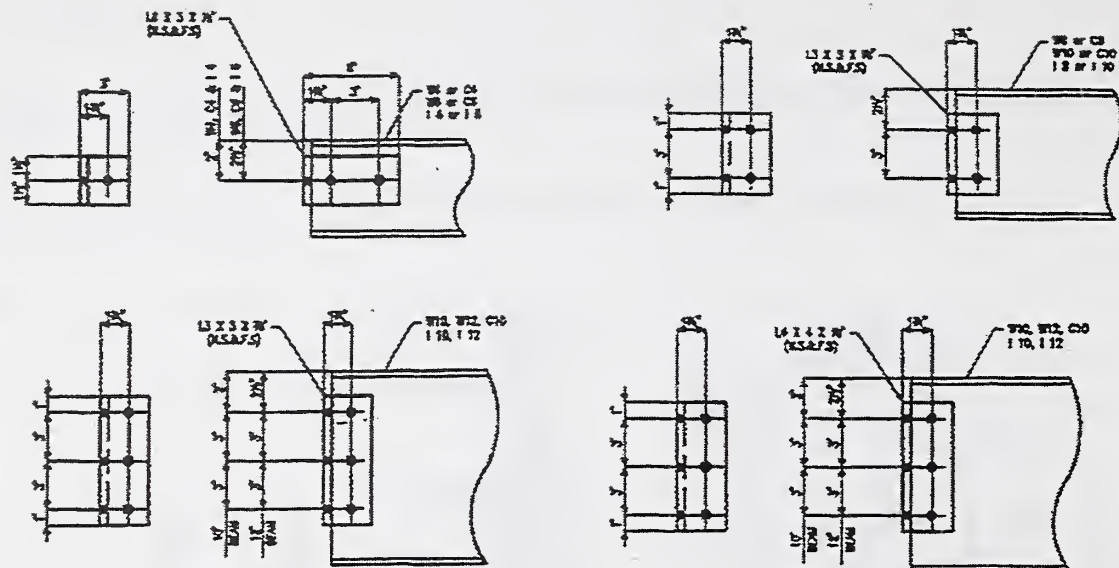
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Chapter 2
Bolted Connections

2.1 INTRODUCTION

The use of mechanical fasteners to join FRP members is a logical extension of their use in joining metallic members. Early 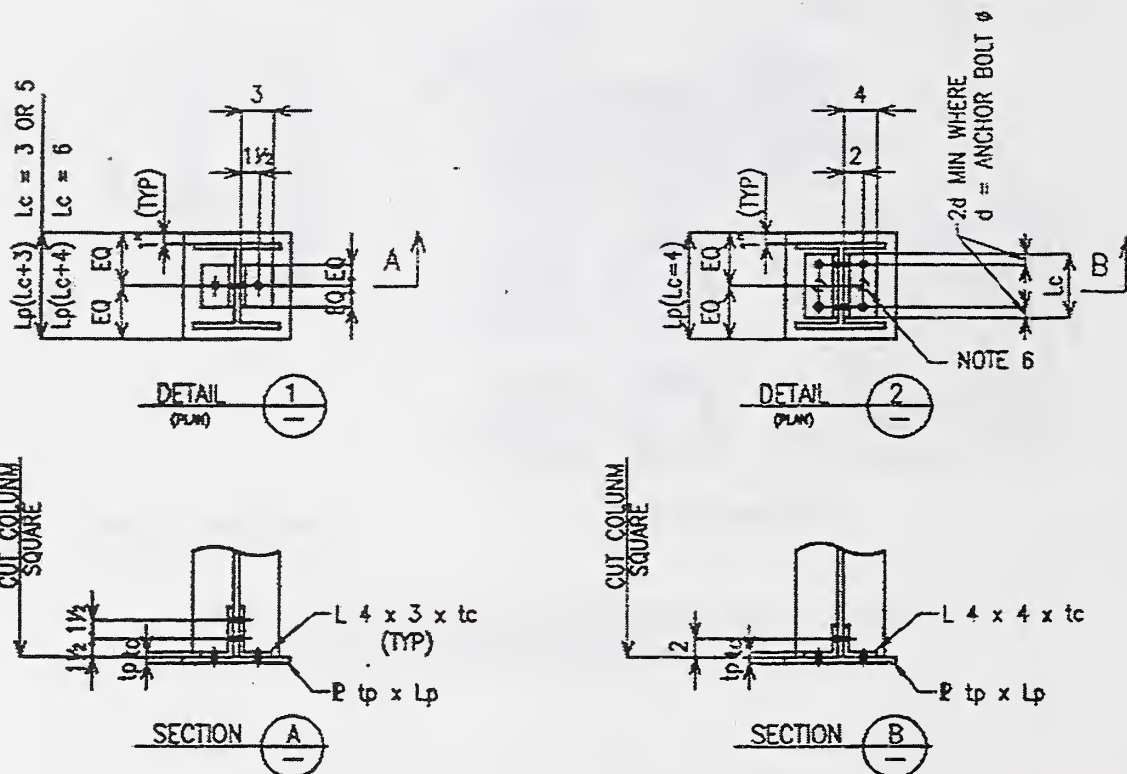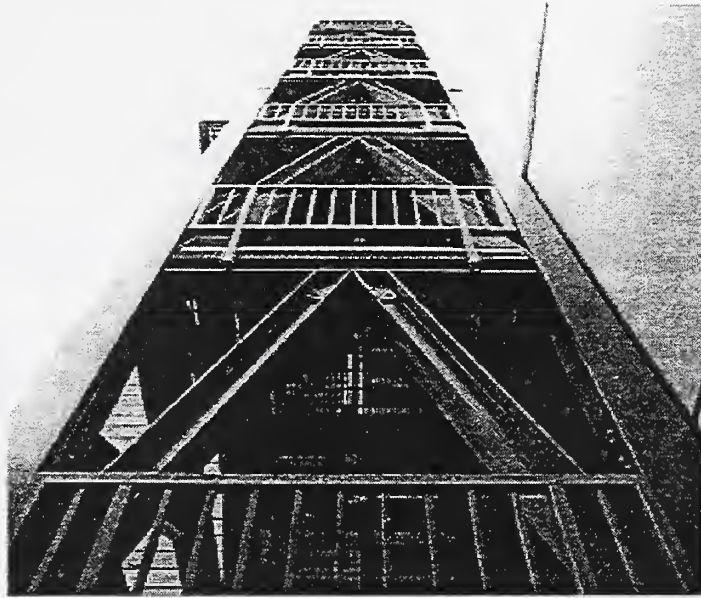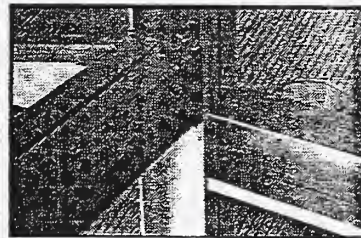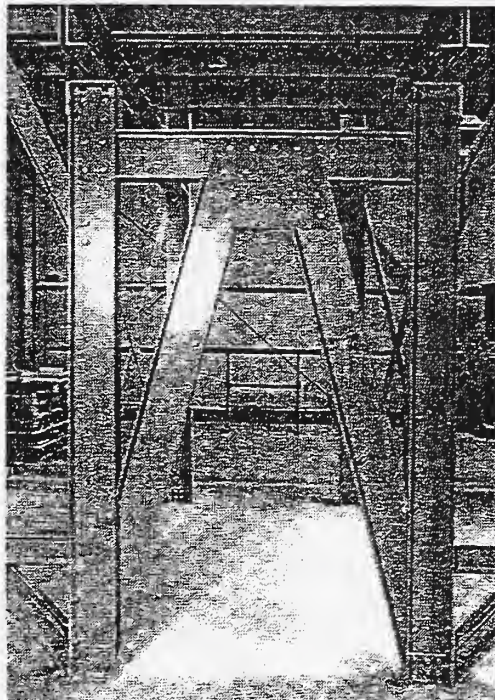on, there was an excessive reliance on design methods originally developed for isotropic materials, as well as a reluctance to punch holes in laminates for fear of cutting the fibers. Researchers have since developed design methods more appropriate to FRP and showed that, although mechanically fastened FRP joints share the same basic failure modes with metals, the mechanisms by which damage initiates and propagates can be fundamentally different and so classical metal failure criteria are not always applicable.

FRP components can indeed be considerably weakened by holes and cut-outs; this can be attributed to the large stress concentration at such discontinuities and to a lack of plasticity. In an infinite, uni-directionally reinforced sheet, the stress concentration at a circular hole can be as large as 8, compared with a value of 3 for isotropic materials (Collings, 1987). Most isotropic materials exhibit some plasticity, which relieves the stress concentration and causes it to have only a small effect on the net failure stress. This is not the case for uni-directional FRP, which is elastic to failure, and where stress concentration causes low net failure stress. The various modes of failure, the design parameters affecting joint performance, and the effect of environmental conditions will be discussed in this brief overview, which mostly covers bolted joints.

2.2 CHOICE OF FASTENERS

Self-tapping screws are perhaps the simplest form of fasteners. They are used where there is no access to the reverse side of a joint, but are inefficient due to the possibility of stripping of the threads cut in the FRP laminates.

Rivets are suitable for joining laminates up to 3 mm (0.1 in) thick. The closing pressure is not always readily controllable, resulting in a wide variation in clamping pressure. In addition, the riveting operation can potentially damage the laminates.

Collings (1977) showed that bolted joints are the most efficient form of all mechanical fastening for carbon FRP (CFRP). On a specific strength basis (strength/mass), they are even superior to conventional structural materials.

2.3 FAILURE MODES

In addition to failing in modes similar to those found in metallic connections, i.e., by shear, tension or bearing, FRP connections may also fail by cleavage of the laminate or the connector pulling through the laminate (Fig. 2.1).

2.3.1 Tension

The average net stress σ_n across a section is an obvious consideration in the design of a joint:

$$\sigma_n = \frac{P}{(w - nd)t} \tag{2-1}$$

where P is the tension carried by the joint of width w and thickness t with n bolt holes of diameter d within the section.

However, stress concentration often causes failure to initiate at the hole. Unlike metals, FRP are linearly elastic up to failure, and consequently no yielding alleviates stress concentrations. However, Hart-Smith (1980) noted that slippage between resin and fibers provides some relief of stress concentration induced around bolt holes and cut-outs (Figs. 2.2 and 2.3). The most highly loaded fibers pull out of the resin over a short length to alleviate high fiber strains, i.e., local debonding or delamination of fibers from resin allows local load sharing between fibers. Thus slippage between resin and fibers averages out local stress concentrations and results in more of the fibers being loaded before final failure. This leads to the surprising conclusion that, in the vicinity of holes, improving adhesion between resin and fibers can in fact lower joint strength. However, reinforcing the vicinity of holes with fibers in different directions reduces the degree of anisotropy, and introduces some pseudo-plastic behavior or softening (Hart-Smith, 1980).

Tensile strength depends highly on fiber orientation. The fibers parallel to the load (0°) carry most of it, and for CFRP, failure is initiated at the stress concentration at the edge of the hole at 90° to the loading axis (Potter, 1978). For glass FRP (GFRP), failure is a more complex combination of shear and tension failures. Failure initiates with the 0° fibers shearing over the projected area of the hole and propagates by in-plane shearing between the 0° and 45° plies across the entire width of the laminate. As a consequence, only the 45° plies remain to carry the axial load, and tension failure occurs immediately after in-plane shearing (Godwin, Matthews and Kilty, 1982).

Aramid fibers (Kevlar) exhibit low lateral adhesion (compared with its high tensile strength) and tend to split along the fiber axis (fibrillation). This relieves the stress concentration at the hole edge and contributes to the tensile strength of the connection (Collings, 1987).

2.3.2 Shear Failure

The shear strength τ of an FRP joint of thickness t carrying a force P is calculated in the same manner as for isotropic materials,

$$\tau = \frac{P}{2et} \tag{2-2}$$

where e is the distance from the center of the bolt hole to the ends of the connected plates. As mentioned above for tension failure, fiber orientation is also critical in determining shear strength. For 0° fibers, the shear strength of a joint is low compared with the in-plane shear strength of the laminate, thus indicating high shear stress concentration around the hole. In contrast, the shear strength of 0°/ 45° joints is high and insensitive to end distance, suggesting low shear stress concentration.

2.3.3 Bearing Failure

Bearing causes compression on the loaded half of the bolt hole. Thus the compressive strength of the 0° fibers, as well as the clamping pressure are important parameters in determining bearing strength (Collins, 1977, 1982; Kretsis and Matthews, 1985; Stockdale and Matthews, 1976; Matthews and Kalkanis, 1985). An average bearing stress assumed to act uniformly on the cross-sectional area of the hole can be calculated,

$$\sigma = \frac{P}{n \, d \, t} \qquad (2\text{-}3)$$

Working with CFRP, Collings (1982) showed that failure initiated in shear at the hole edge, through both fibers and matrix. Laminates containing some 90° and / or ± 45° plies perform well under bolt bearing conditions. This is contrary to what might be expected from the compressive behavior of 90° and ± 45° laminates, thus demonstrating that bearing failure mechanisms are different from those of compressive failure. The difference is due to the lateral pressure exerted by the clamping action of the bolt. Constraint against Poisson expansion normal to the plane of the laminates causes the fibers transverse to the load to fail in constrained transverse compression.

Kretsis and Matthews (1985) suggested that the low elastic modulus of glass fibers favors an instability mechanism that is not fully prevented by lateral constraint. Matthews and Kalkanis (1985) showed that the fibrillar nature of Kevlar fibers causes weak planes of failure. Bearing strength thus depends more on this weak bonding between "fibrils" than on the interface between fibers and matrix.

2.3.4 Cleavage Failure

Cleavage failure only occurs in 0 ° / ± θ lay-ups with a high proportion of 0 ° fibers (θ is the orientation of the plies that are not at 0 °). Failure initiates in a single shear mode followed by failure of the net section on one side of the laminate. If such lay-ups are needed, the region around the hole should be reinforced.

2.3.5 Pull-out Failure

Pull-out failure is associated mostly with rivets. Since rivets are used mostly in single shear, axial in-plane loading imposes out-of-plane bending and peeling (pulling apart) of the joint. Under extreme loading, rivets may fail by bending or head shearing.

Of the above modes of failure, the most desirable are bearing and tension (Fig. 2.4).

2.4 IMPORTANT FACTORS AFFECTING JOINT STRENGTH

2.4.1 Fiber Orientation

As mentioned in the discussion on modes of failure, fiber orientation has a strong effect on joint strength. Collings (1977) showed that the best overall joint performance for CFRP is obtained

for a combination of 0° / ± 45° plies. Figure 2.5 shows that, at low levels of ± 45° fibers, shear failure predominates. As the number of ± 45° fibers increases, so does shear strength, until bearing becomes the critical mode of failure. Since ± 45° laminates are weak in tension, further increases in the proportion of ± 45° fibers changes the failure mode to tension.

2.4.2 Lateral Constraint

Lateral constraint due to clamping pressure can significantly increase joint strength, although. over-tightening of bolts may cause surface damage to the laminate. Collings (1977) and Garbo and Ogonowski (1981) recommended an optimum bolt clamping pressure of 22 MPa for CFRP.

Resin creep causes clamping pressure to decrease in time (Shivakumar and Crews, 1982). This behavior is more pronounced in high temperature and high moisture conditions (up to 60% clamping pressure loss at 60 °C, 1 % moisture content). This loss in clamping pressure does not necessarily translate in an equal loss in joint strength. Indeed, finger-tight constraints produce joint strength of 88 % of optimally constrained joints for dry, 0°/ ± 45° laminates at room temperature (Collings, 1982). See Figs. 2.6-2.8.

2.4.3 Stacking Sequence

Collings (1977) showed that, for bolted CFRP joints consisting of 0° (2/3) and ± 45° (1/3) plies, there was no difference in shear strength, but a difference of 6 % in tensile strength between two different stacking sequences. Bearing strength, however, shows a significant drop (16 %) for the more "grouped" laminates (Fig. 2.9, Table 2.1).

For pin-loaded holes (without clamping pressure), Quinn and Mathews (1977) showed that a 90°/±45°/0° laminate produced the highest bearing strength, and a 0°/90°/±45° the lowest, with a relative reduction of 30 % (Fig. 2.10).

2.4.4 Joint Geometry

Width: In contrast to metals, FRP cannot take advantage of plasticity to relieve stress concentration near bolt holes. As a result, for a given hole diameter, net tensile failure stress depends strongly on width. The effect of width is most marked for laminates with a high proportion of 0° fibers and least for ± 45° laminates (Collings, 1977, Figs. 2.11 and 2.12).

Hole Size: Hole size has little influence on the net tensile strength and shear strength of 0°/ ± 45° CFRP (Collings, 1977) or 0°/ ± 45° GFRP (Kretsis and Matthews, 1985). The bearing strength of CFRP is unaffected by hole diameter d provided sufficient clamping pressure is provided. However, for GFRP, the low elastic modulus of glass causes out-of-plane cracking for $d/t > 3$ regardless of lateral constraint, in 0°/ ± 45° lay-ups. Values of $d/t < 1.5$ should not be used due to the risk of bolt shear. Figures 2.13 - 2.15 show the effect of hole size, non-dimensionalized with respect to plate thickness t or width w. Joint strength is limited by bearing failure for small d/w ratios, and by tension failure through the hole for large d/w ratios. A reduction in end distance e, below a certain value of $e/w \approx 1$ further decreases joint strength.

Interaction between holes: Joint geometry is generally chosen such that potential tensile or shear failures occur simultaneously at a mean stress as close as possible to the bearing failure stress. If these requirements are met, and adequate spacing is provided for ease of installation, then interaction between adjacent bolts and holes is minimal. On the other hand, matching the stiffness of the joining parts equalizes the load distribution to the bolts and maximizes joint strength (Fig. 2.16).

2.5 EFFECTS OF TEMPERATURE AND MOISTURE

Exposure of FRP to a wet environment allows the absorption of moisture into the resin matrix, resulting in plasticization and a lower glass transition temperature. Joint properties most affected are shear and bearing strengths. Kim and Whitney (1976) showed that temperature had a more significant effect than moisture on bearing strength. The combination of temperature and moisture produces a further loss of bearing strength of 10 % compared with the effects of temperature alone. They measured a maximum bearing strength reduction of 40 % in the case of combined temperature and moisture.

2.6 FATIGUE STRENGTH

The fatigue performance of bolted CFRP joints can be better than that of open holes. Thus mechanical joints of composite materials are unlikely to be fatigue critical, in contrast with aluminum alloys (Clayton and Jones, 1976; Heath-Smith, 1979). Garbo and Ogonowski (1981) showed that residual strength is generally equal to or greater than the strength of non-fatigued specimens, because of progressive relief of stress concentration. However, in most cases, the specimens sustained hole elongations of 0.05 d, a value greater than that normally allowed in metallic joints. Therefore, elongation, rather than strength, may be the governing fatigue criterion for CFRP joints.

2.7 CONCLUSION

Mechanical fasteners are an effective method of joining FRP members. The best overall performance for the three main failure modes (tension, shear and bearing) is obtained for 0° / ± 45° laminates. As a general rule, there should be between 1/8 and 3/8 of the fibers in any one of the basic laminate directions, 0°, + 45°, - 45° and 90° (Hart-Smith, 1987). Maximum joint strength is developed when the joint geometry is designed to suppress tension and shear failures, and sufficient clamping pressure across the thickness of the laminate is provided.

2.8 FUTURE RESEARCH

The EUROCOMP Design Guide and Handbook (1996) provides general design procedures for mechanical connections loaded in shear or tension, as well as a more rigorous finite-element method. The Guide recommends designing other mechanical joints by testing. Clearly, further research is needed to define design procedures for all mechanical joints.

The use of thicker plies, or the stacking together of thinner plies significantly reduce laminate strengths, a fact not accounted for in most laminate theories. Since civil engineering applications generally use thicker members than aerospace engineering does, more research in the joining of thicker elements is necessary. The development of stronger and tougher resins is also considered the key to better joints (Hart-Smith, 1978).

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TENSION FAILURE

SHEAROUT FAILURE

BOLT PULLING THROUGH LAMINATE

CLEAVAGE-TENSION FAILURE

BEARING FAILURE
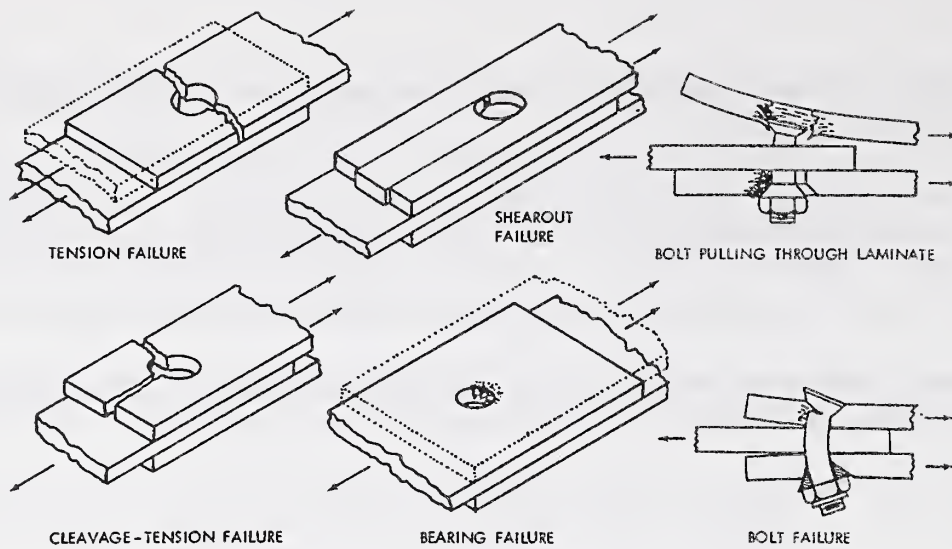
BOLT FAILURE

Figure 2.1 Modes of failure for bolted joints in
FRP composites (from Hart-Smith, 1987)

σ

NO DELAMINATION

DELAMINATION

NO CHANGE IN FIBER STRESS THROUGHOUT DELAMINATED REGION

HIGH PEAK TENSION STRESSES IN FIBER ADJACENT TO HOLE

REDUCED PEAK FIBER STRESS BECAUSE STRAIN SPIKE IS AVERAGED OVER FINITE LENGTH

STRESSES IN FIBERS ALONGSIDE HOLE

σ
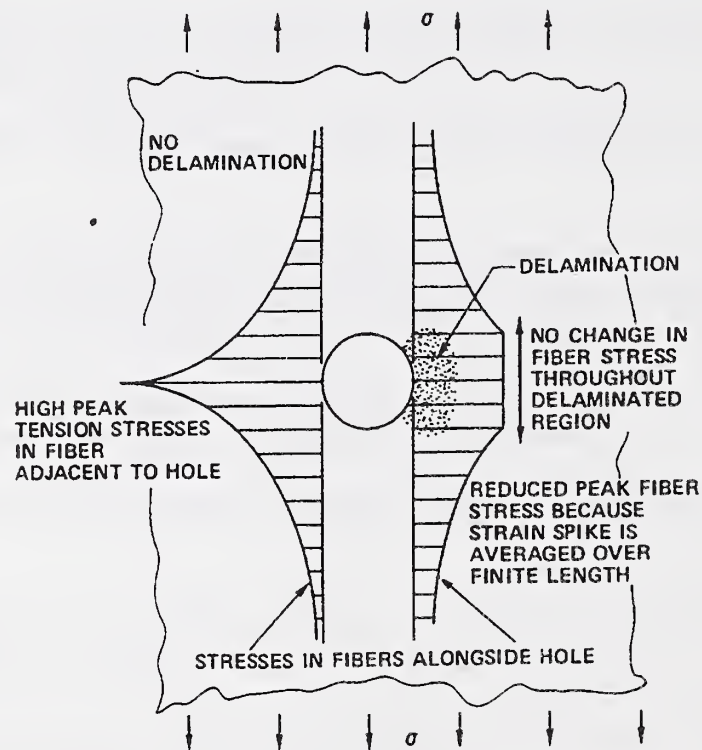
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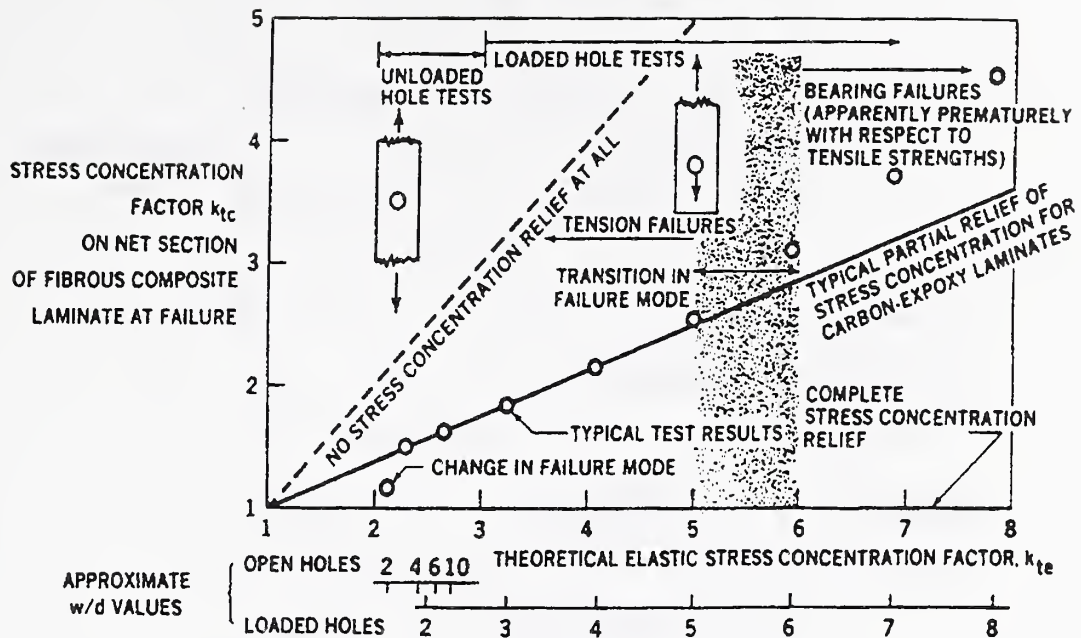
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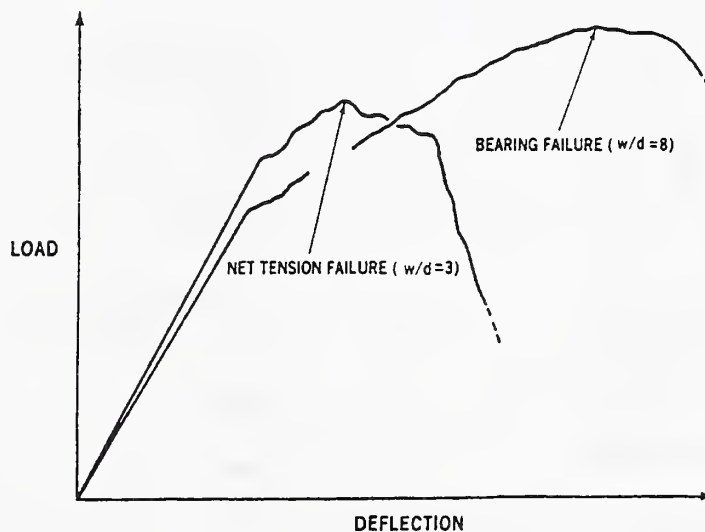
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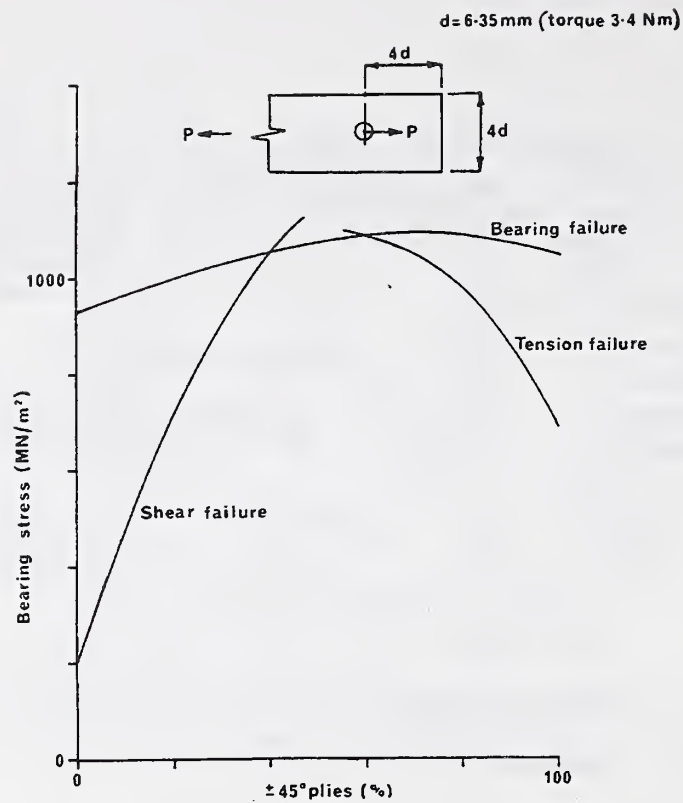
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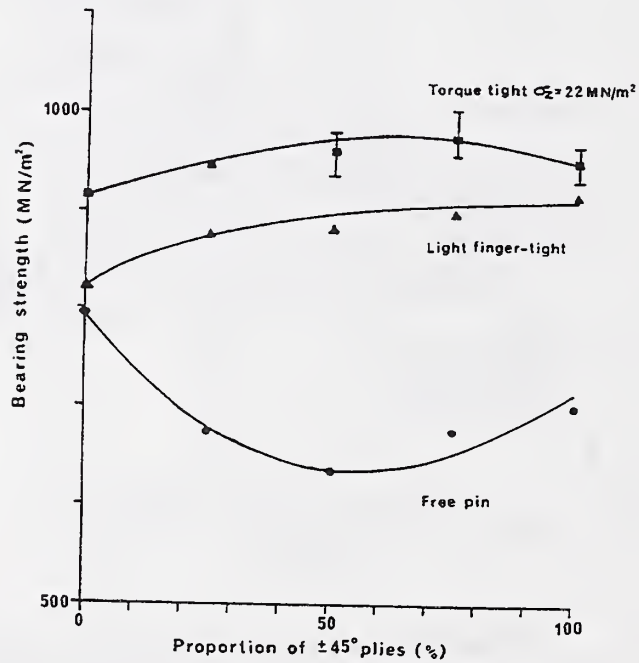
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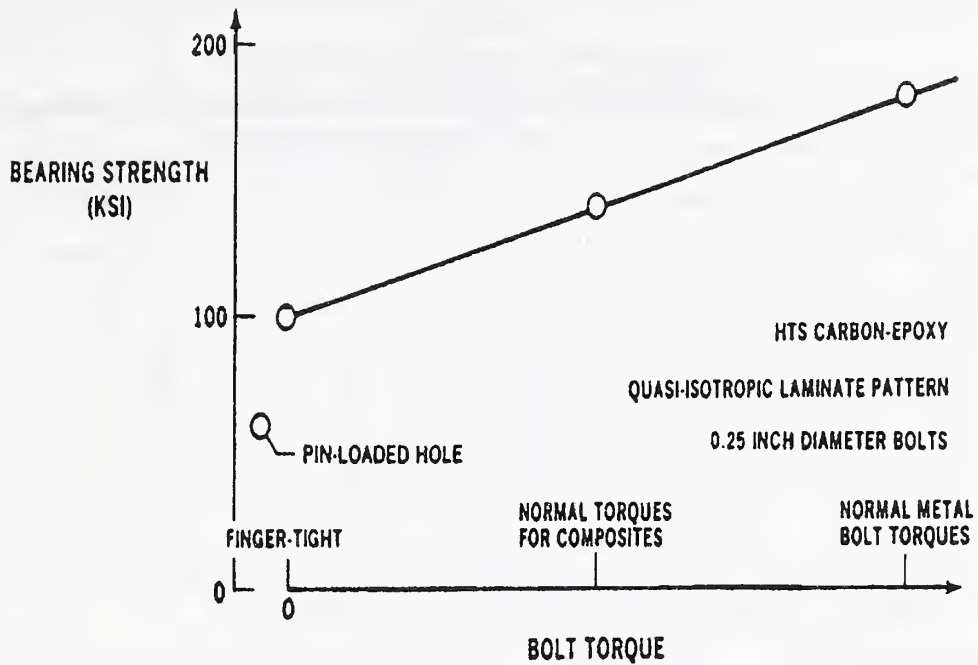
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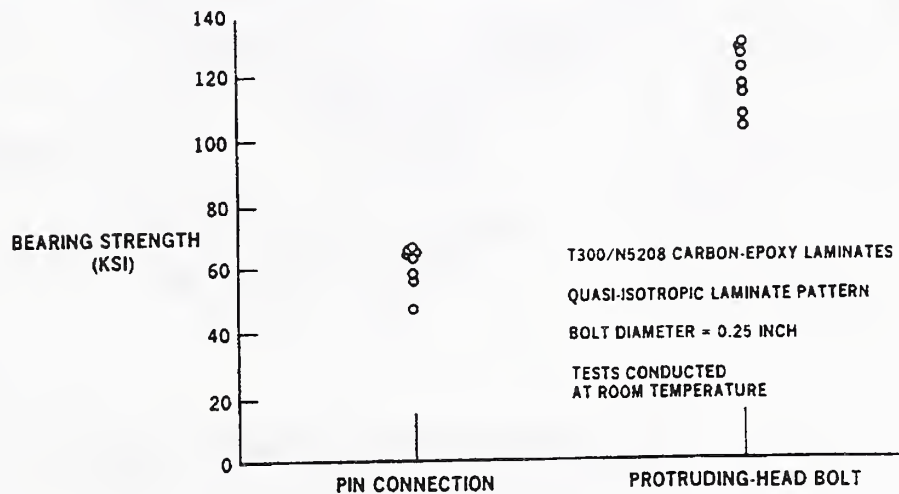
Fig 2.8 Comparison between bearing strengths of connections with and without clamping pressure (100 ksi = 690 MPa, 1 in = 25 mm) (from Hart-Smith, 1987)

Ply number to centreline[b]	Ply orientation (degrees)[a] Lay-up number				
	1	2	3	4	5
1	+	+	+	+	+
2	0	-	-	-	-
3	-	0	+	0	0
4	0	0	-	0	0
5	90	+	90	90	0
6	0	90	0	0	0
7	+	-	0	+	0
8	0	0	0	-	+
9	-	0	0	0	-
10	0	0	0	0	90
Number of plies	20	20	20	20	20
Percentage of 0/±45/90°	50/40/10	50/40/10	50/40/10	50/40/10	50/40/10

[a] + and - refer to ±45° ply orientation.
[b] Lay-ups are all symmetric about the centreline.
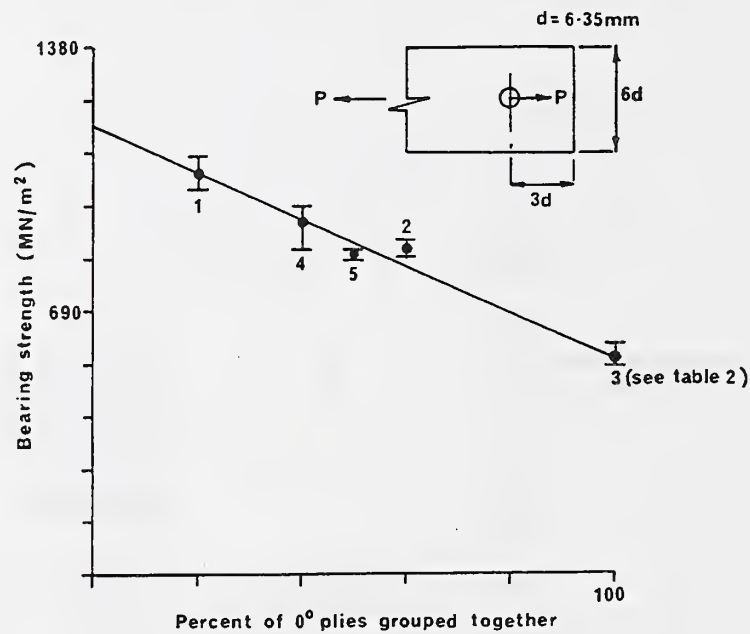
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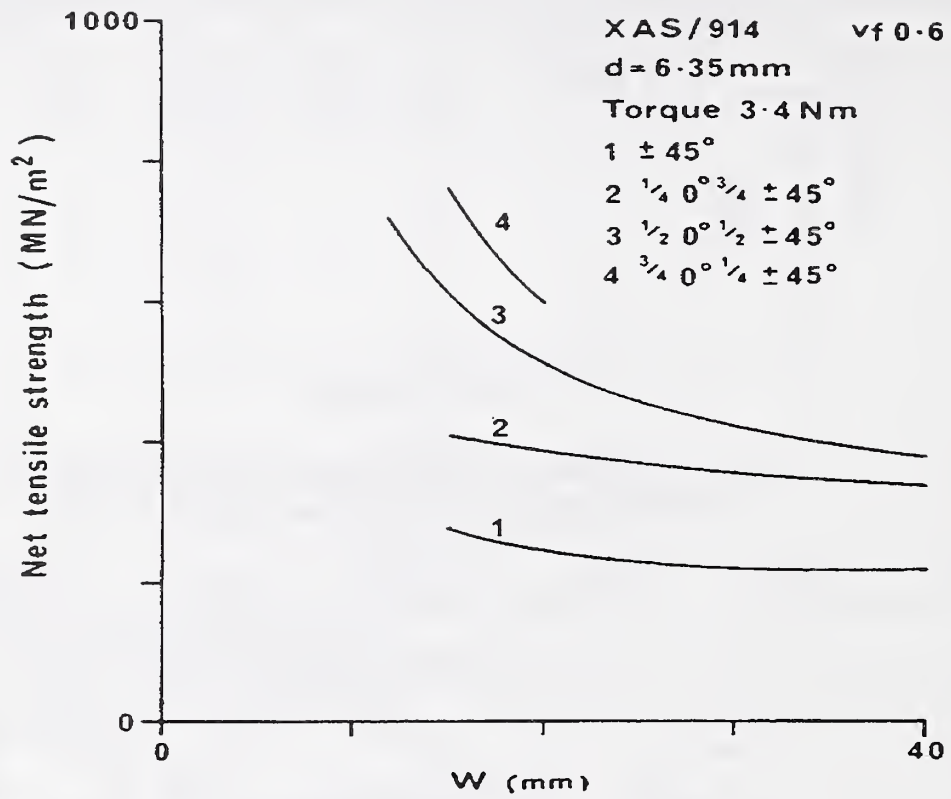
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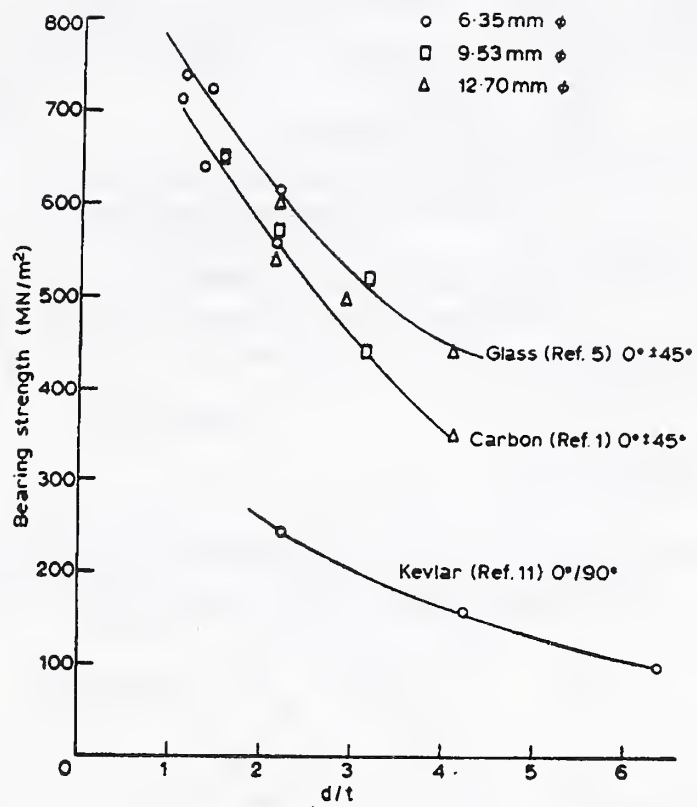
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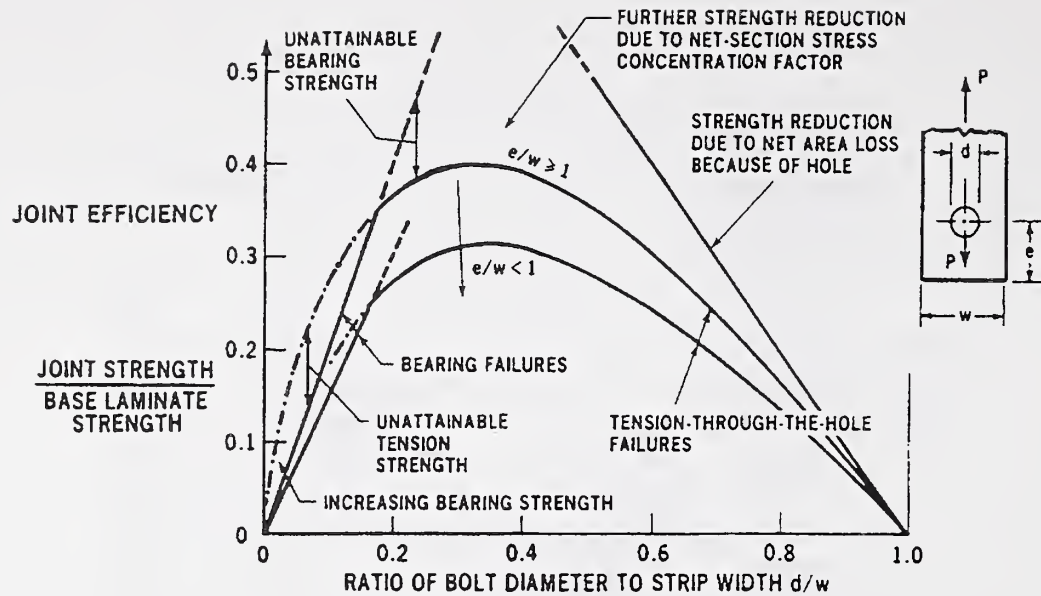
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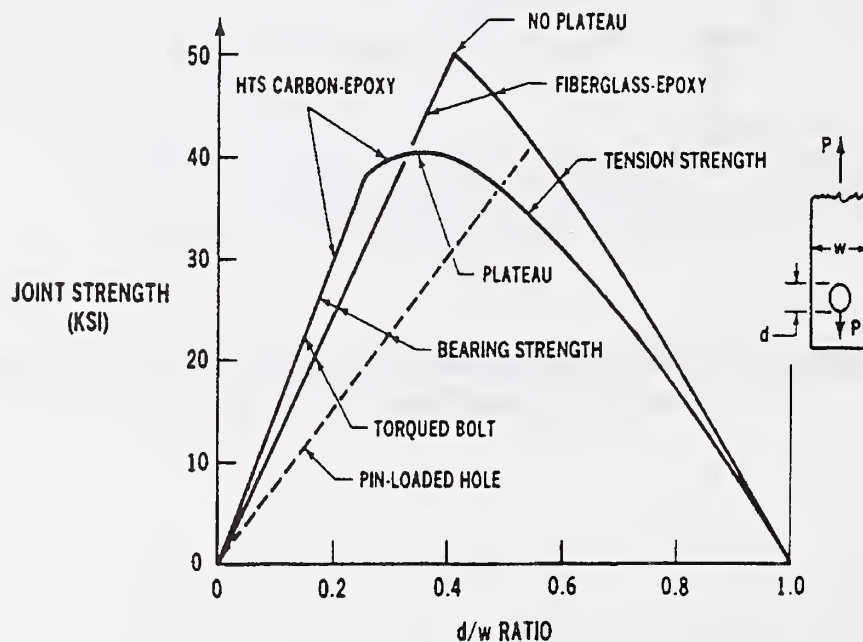
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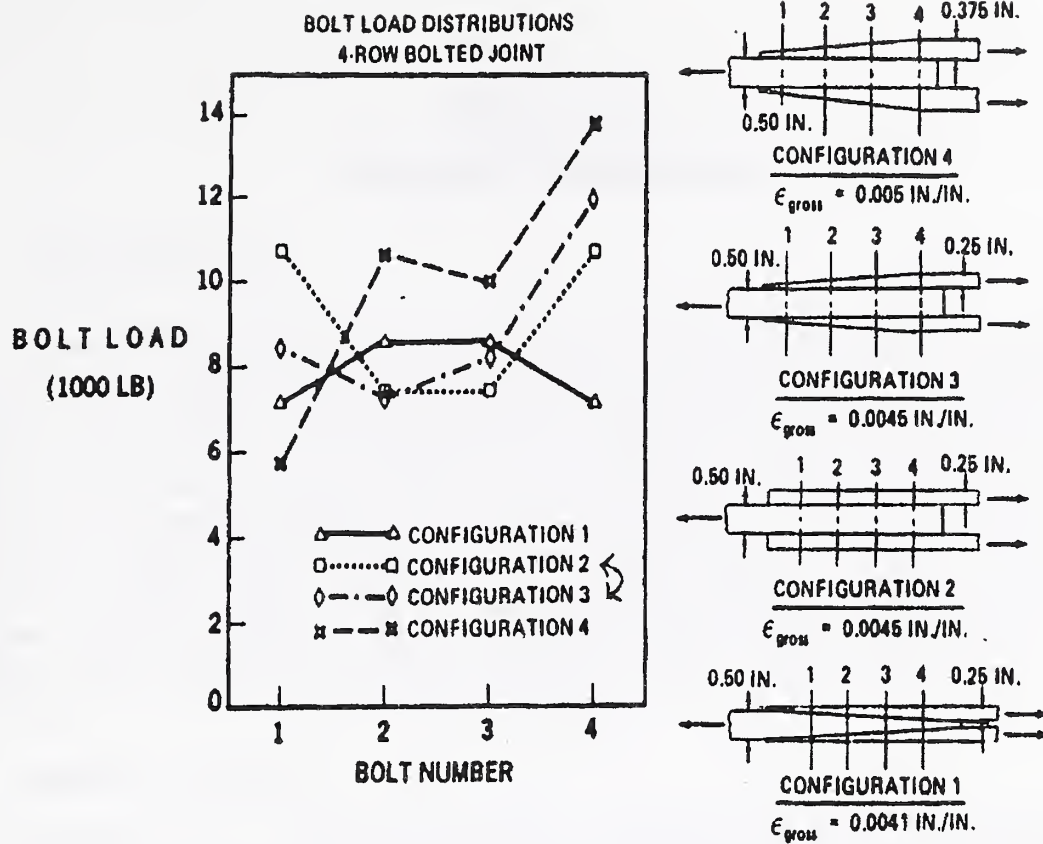
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Chapter 3
Bonded Connections

3.1   INTRODUCTION

As discussed in the previous chapter, FRP members can be joined by bolts or rivets. However, these mechanical fasteners sometimes require the use of load-spreading inserts bonded to the structure (Fig. 3.1, top). An alternative method of joining is by bonding using adhesives. An advantage of bonded joints is the possible use of co-curing techniques, in which the composites are joined without using a separate adhesive. In this process, the composites are prepared in their uncured (pre-impregnated fiber) form and heat and pressure are applied. As the composite begins to cure, the pressure squeezes out the excess matrix material, which contacts the adjacent component. When curing is complete, the components are bonded together. However, this technique would be difficult to apply to large-scale civil structures, which tend to be field-assembled. This overview of bonded joints covers stress analysis, failure modes, design implications and the effects of the operating environment.

3.2   STRESS ANALYSIS

With the assumption that the adherents are rigid and the adhesive deforms elastically and only in shear, a simple stress analysis of an adhesively bonded, single-lap joint produces a uniform shear stress distribution in the adhesive. While this simple analysis is approximately true for a short overlap, Volkersen's (1938) more refined analysis, which accounts also for the elastic deformation in tension of the adherents, shows shear stress concentration at the ends of the connection (Fig. 3.2).

 Goland and Reissner (1944) accounted for bending effects due to load eccentricity in a single lap joint (Fig. 3.3). In an undeformed lap joint connecting two plates of thickness t, the moment due to eccentricity of load P is $M = P t / 2$. The deformation of the joint reduces this moment by a factor $K<1$. Analysis also shows the presence of transverse normal tensile stresses that cause peeling near the ends of the joint.   Since modern adhesives can be stronger than the polymer in FRP, this normal stress can cause tensile failure in the adherents if the interface is also strong.

More recent analytical refinements include: zero stress boundary conditions at the ends of the adhesive, non-uniform shear and normal stress distributions across the thickness of the adhesive, shear deformation in the adherents, and stiffness of the adhesive (Kutscha and Hoffer, 1969; Renton and Vinson, 1975; Allman, 1977). A more flexible adhesive produces more uniform shear and normal stress distributions than a stiffer one (Fig. 3.4), the latter being sometimes required to limit deformations. Adams and Peppiatt (1973) showed the existence of significant shear stresses in the adhesive and normal stresses in the adherents in the direction transverse to the applied loads, across the width of a lap joint. According to Matthews et al. (1982), it is necessary to consider non-linear adhesive behavior in order to correctly predict joint strength. In spite of recent advances, numerical models are no substitute for testing and experience, which are still necessary, especially for a joint that is frequently used.

3.3   FAILURE MODES

Adhesively bonded joints can fail in three distinct modes (Hart-Smith, 1987):

- The strongest joints do not fail in the adhesive at all. Rather, they fail outside the joint area at 100 % of the adherent *tensile* strength. The strength of the joint is thus proportional to the laminate thickness *t*. However, *delamination* of fibrous composite adherents is a considerably weaker mode of failure.
- The next highest strength is limited by the shear strength of the adhesive. The strength of the joint is here proportional to the square root of the laminate thickness.
- The weakest failure mode is associated with failure of the adhesive under peel stresses. The peel strength is proportional to the quarter power of the laminate thickness.
- In addition, failure of the interface between adhesive and adherents is a possibility.

Adhesives are most efficient only in the thickness range of 0.125 mm to 0.25 mm (0.005 in to 0.010 in). Figures 3.5 and 3.6 show the failure modes and strength envelopes of adhesively bonded single-lap joints. Depending on the coefficient of proportionality between the various strengths and the powers of *t*, the strength envelope can be more or less complicated. Fig. 3.7 shows the peel stress failure of thick composite joints.

3.4   DESIGN IMPLICATIONS

Due to the non-uniform shear stress distribution in the adhesive, extending a lap joint beyond a certain length does not increase its strength, but does provide safety against creep rupture, provided the stress in the central part of the joint is sufficiently low (Figs. 3.8 and 3.9). The stress trough in the middle of a lap joint should thus not be regarded as an inefficiency to be designed away. In fact, Hart-Smith (1982) considers that limiting the *minimum* strain (to 10 % of the maximum at ultimate load) in a lap joint is more important than limiting the *maximum* strain.

A symmetrical design, such as a double lap joint, alleviates bending effects to some extent, but load eccentricity and bending are still present, thus leading to tensile peeling stresses (Fig. 3.10). The addition of a fillet at the end of the lap is very effective in reducing stress concentration and increasing joint strength (Fig. 3.11). Other designs, such as tapering the end of the adherents, accompanied by local thickening of the adhesive, reduce stress concentration for both shear and peel stresses, but do not make as significant an improvement in joint strength as a well designed fillet does (Fig. 3.12).

A stepped lap joint is an attempt to render the shear stress distribution more uniform and allows an efficient use of a long lap (Fig. 3.13). Each step, in effect, functions as an individual lap joint, with the butt faces at the end of each step transferring a significant portion of the load. An important design consideration is the need to balance adherent stiffness at both ends of the joint. The joint on the right in Fig. 3.14 is almost twice as strong as the one on the left, yet uses less material!

Another particularly efficient design is a scarf joint, which causes stresses to be much higher at the compression end (where the center adherent ends) than at the tension end of the joint (where the outer adherents end). Tapering of the adherents provides little advantage unless the scarf joint ends in a knife-edge, which is difficult to achieve in practice. A tip thickness of only 0.025

mm (0.001 in) causes a stress concentration of 25 % in the elastic adhesive shear stress distribution (Thamm, 1976). Stiffness balance of the adherents is an important consideration here as well. Figures 3.15 and 3.16 show a variety of bonded joint configurations and their relative strength.

3.5   ENVIRONMENTAL EFFECTS
The performance of adhesives is, in general, highly affected by temperature and humidity. As with all polymers, the stiffness and strength of an adhesive drop sharply beyond the glass transition temperature (Figs. 3.17 and 3.18). The material is rubbery above the glass transition temperature, and glass-like below it. Absorption of water plasticizes adhesives and lowers their mechanical performance (Figs. 3.19 and 3.20). For joints sustaining permanent loads, the visco-elastic properties of polymers must be taken into account.

3.6   SURFACE PREPARATION
For surfaces to bond properly, they must be free of grease, dust, moisture or other release agents. For most applications, surface preparation simply involves solvent degreasing, abrasion and dust removal. FRP surfaces should be abraded until they lose their gloss or luster and become dull. However, one should be careful not to break reinforcing fibers near the surface.

A peel ply is sometimes used if a very clean and reproducible surface is needed for bonding on site. It is a sacrificial layer of fabric laid up on the outermost surface and co-cured with the FRP. This layer is peeled off prior to bonding, leaving behind a clean, rough surface for bonding.

Corona discharge treatment is a factory technique ideal for GFRP skins of large sandwich panels. It makes use of a high frequency electrical discharge, which ionizes atmospheric oxygen and produces a highly oxidized, polar surface with increased surface free energy.

3.7   SUMMARY
Adhesive joints can be highly effective when large surfaces are available for joining and the members are thin. Careful design is necessary to minimize stress concentrations and peeling stresses. A trade-off must usually be made between strength and deformation requirements.

Adhesive joints have been used in structural applications, but usually under tightly controlled factory conditions, as found in the aerospace industry. The bonded joints must be designed to withstand environmental conditions found in their operational life, and frequent inspections must be scheduled to ensure their integrity.

In civil infrastructure, adhesive joints are usually not relied upon as the sole means of connection, mostly because of the thickness of the members to be joined, and the difficulty in controlling quality in field assembly. In cases where members are fastened together simultaneously by bolts and adhesive, joint strength is calculated based on the bolts only, whereas the bond provides additional stiffness and safety. For thin and moderately loaded structures, adhesive bonding provides a fail-safe load path to protect the structure in the vicinity of the fastener holes. On the other hand, thick, highly loaded, adhesively bonded structures need mechanical fasteners to protect the bond from unzipping catastrophically from any small initial damage.

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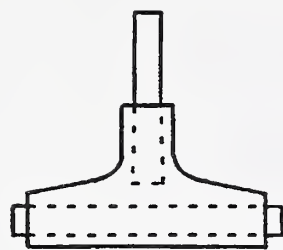
LAMINATED JOINT
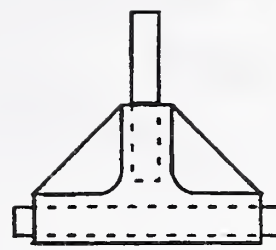
MOULDED JOINT

AXIAL CONNECTIONS

BONDED OR LAMINATED DOUBLERS

LAMINATED SCARF JOINT

BONDED OR LAMINATED SCARF JOINT
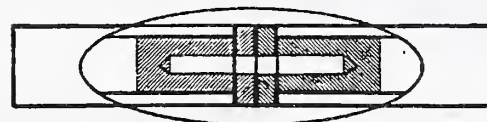
BONDED SCARF JOINT

BONDED INSERT JOINT

BASEMENT/SECTION CONNECTIONS
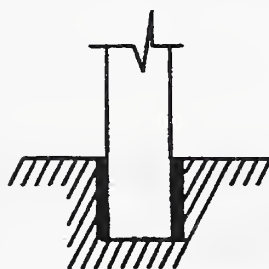
CAST-IN JOINT
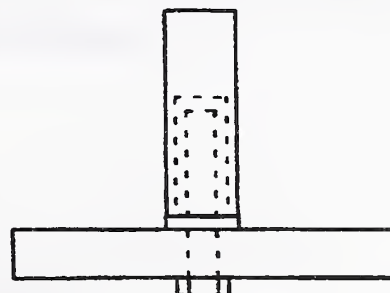
BONDED INSERT/BASE PLATE CONNECTION

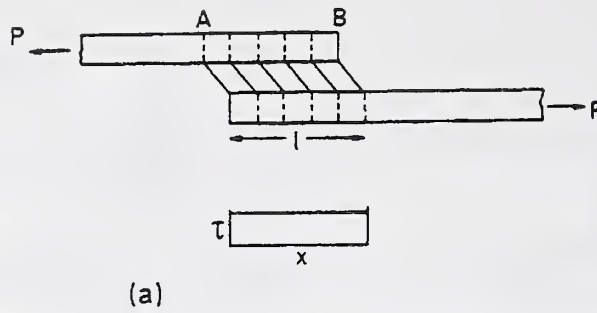Figure 3.1   Joining techniques for pultruded sections.
(from Clarke, 1996)

(a)

(b)
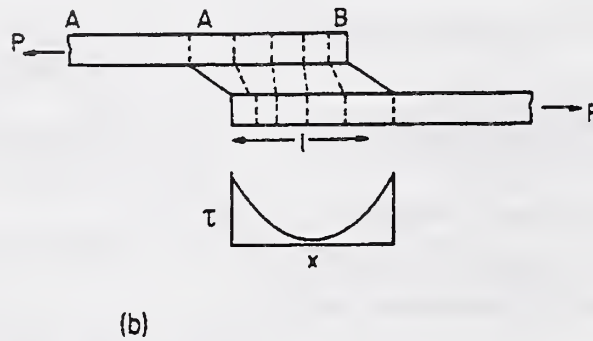
Figure 3.2   Exaggerated deformations in loaded single lap joint:
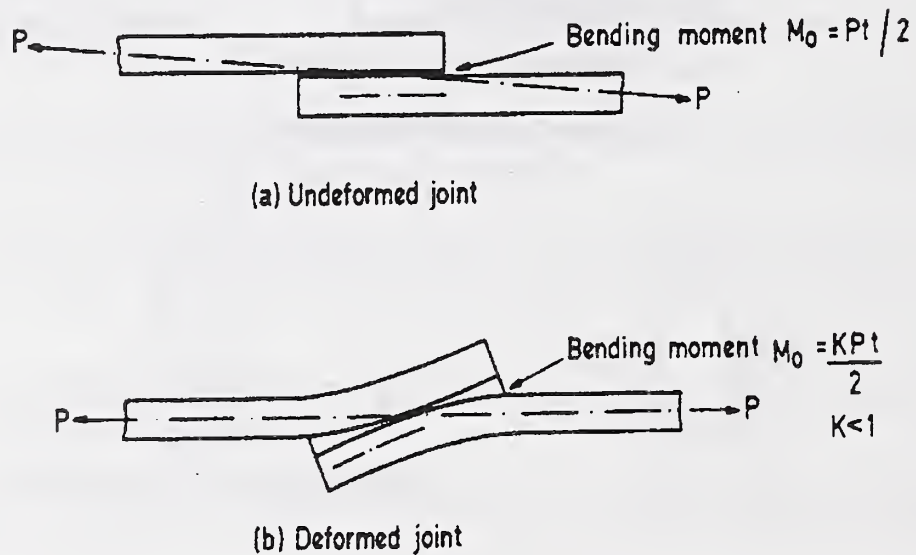(a) with rigid adherends; (b) with elastic adherends.
(from Adams, 1987)

Bending moment $M_0 = Pt/2$

(a) Undeformed joint

Bending moment $M_0 = \dfrac{KPt}{2}$

$K < 1$

(b) Deformed joint

Figure 3.3 A geometrical representation of the Goland and
Reissner (1944) bending moment factor (t = adherent thickness).
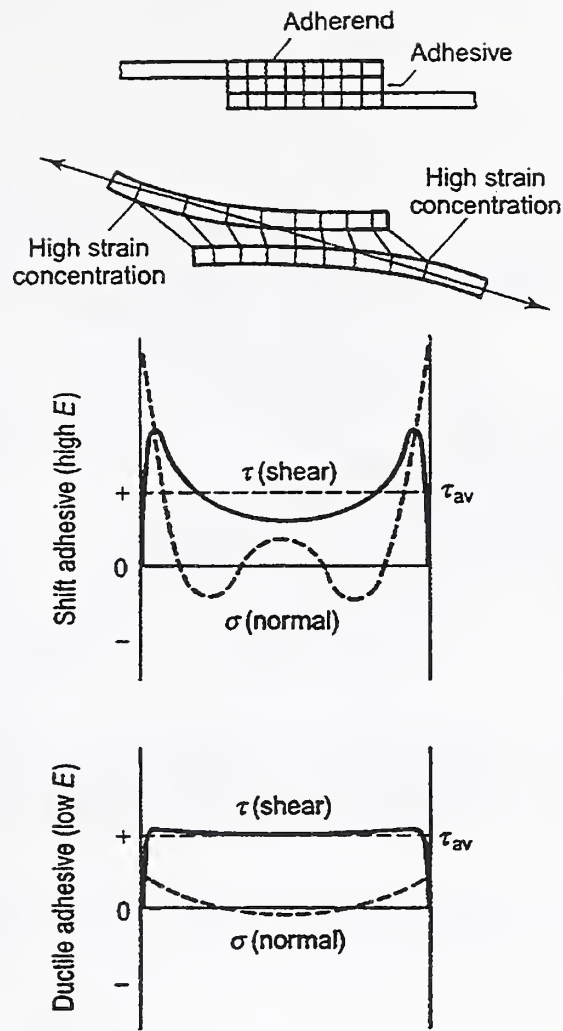
Figure 3.4   Typical elastic bondline stress distributions in a single lap joint
under load, made with adhesives of low and high elastic modulus.
(from Hutchinson, 1997)
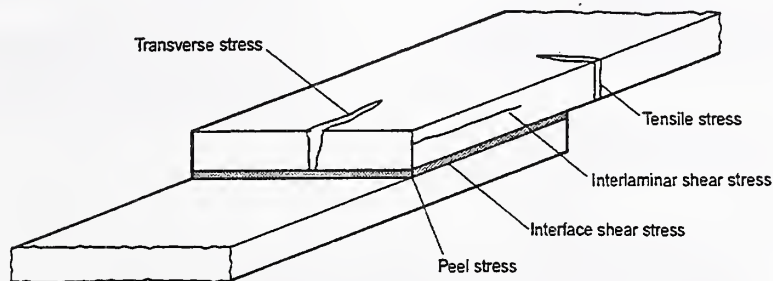
Figure 3.5 Failure modes of single-lap bonded joints
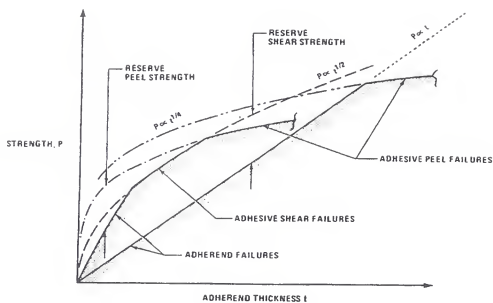
Figure 3.6 Relative severity of adhesive shear and peel
stresses in single lap-joints (from Hart-Smith, 1987).
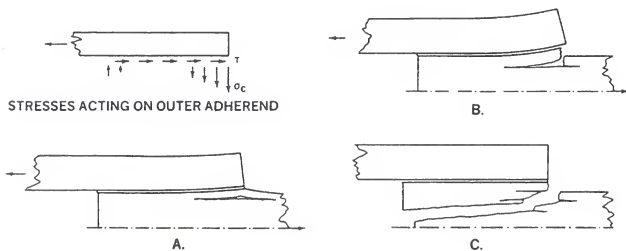
Figure 3.7 Peel stress failure of thick composite
joints. (A), (B) and (C) indicate failure sequence.
(from Hart-Smith, 1987)
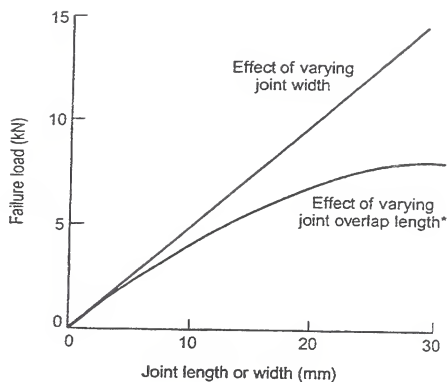
Figure 3.8   Effect of overlap length and width on single lap joint strength (from Hutchinson, 1997).
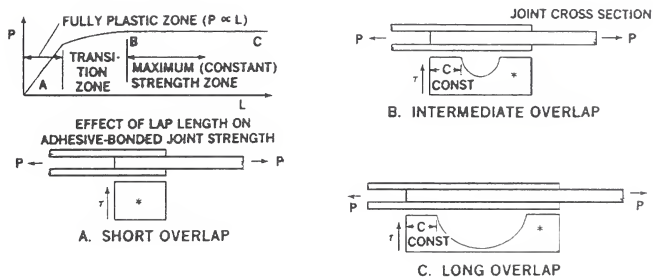
Figure 3.9   Influence of lap length on bond stress distribution. *Adhesive bond stress distribution (from Hart-Smith, 1987)
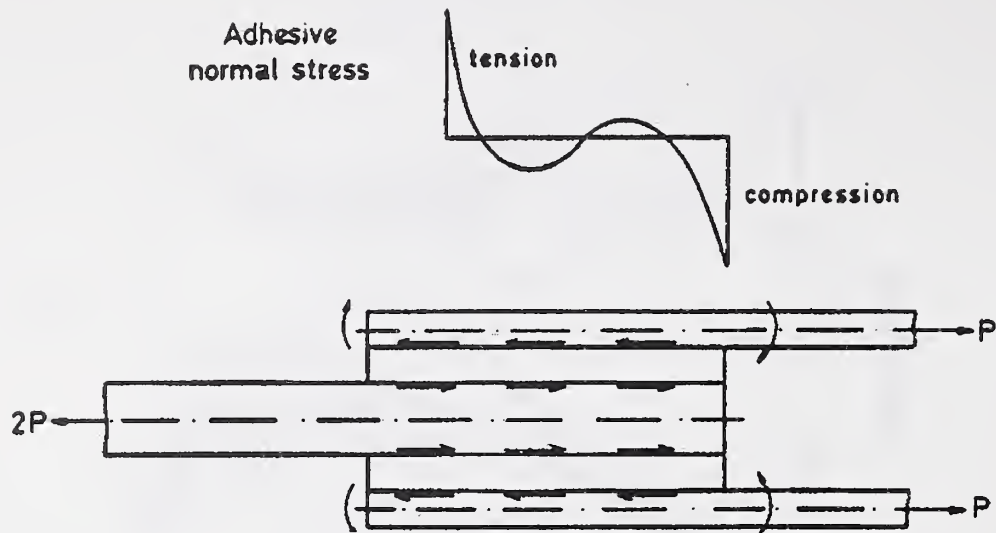
Figure 3.10 Bending moments and induced stresses in the outer adherends of a double lap joint (from Adams, 1987).
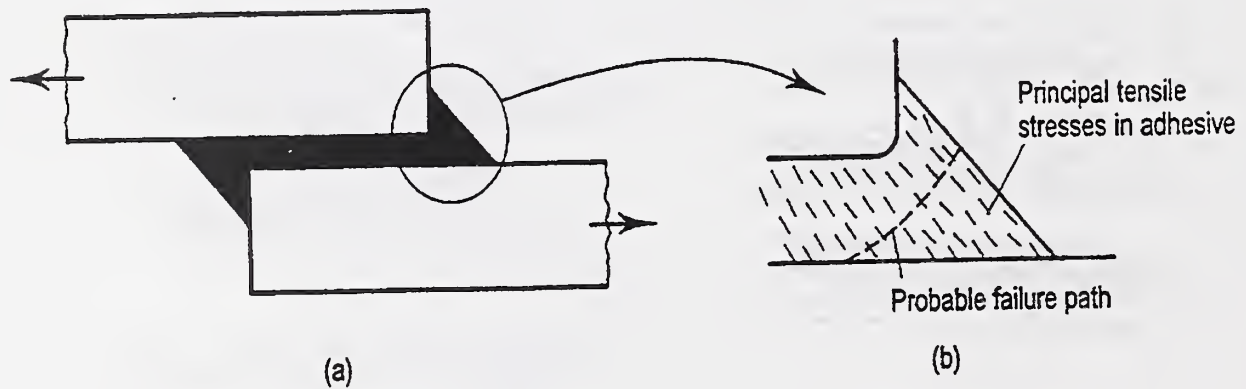
Figure 3.11   Lap joint formed with spew fillet (from Hutchinson, 1997).
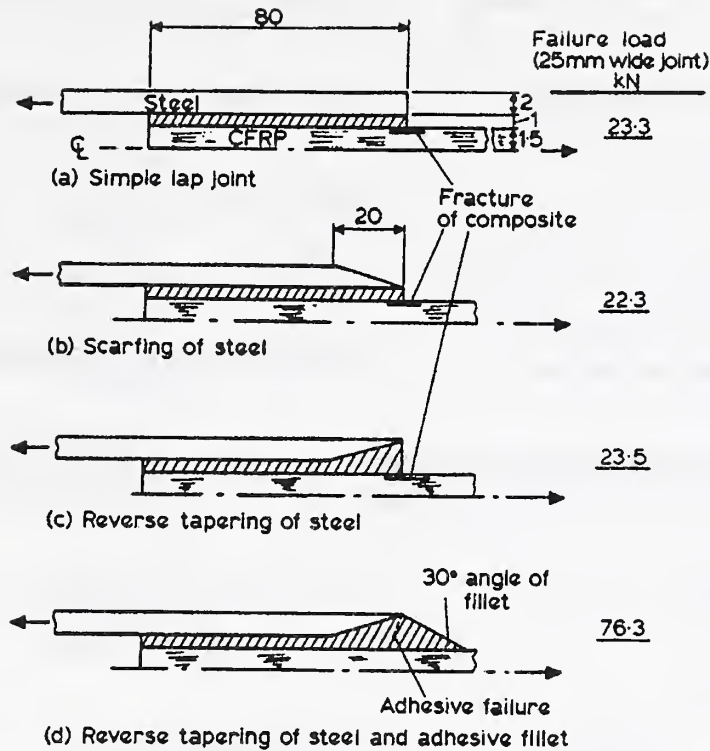
Figure 3.12 Bonding of steel to CFRP (length in mm) (from Adams and Wake, 1984). Fracture of composite caused by through-thickness tensile stresses.
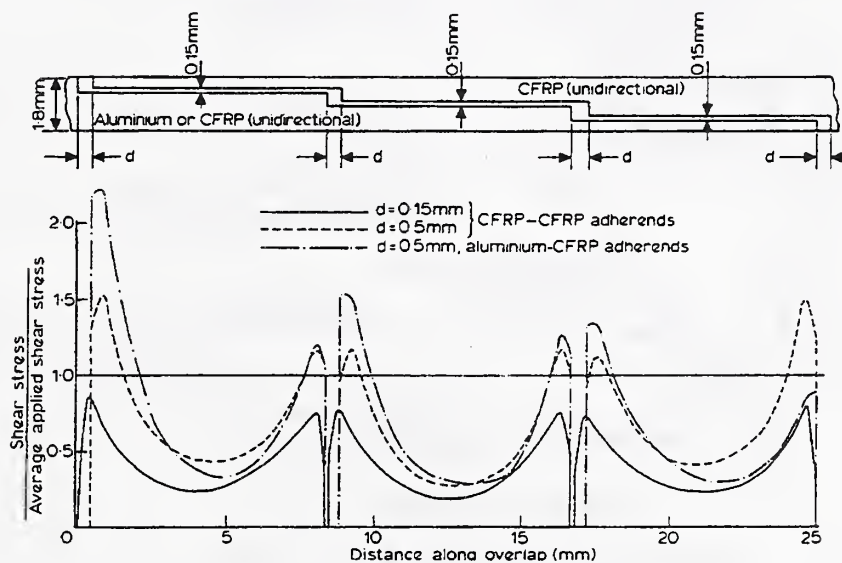
Figure 3.13 Adhesive shear stress distributions in step joints (from Adams and Wake, 1984).

CONSTANT·THICKNESS (STIFFNESS-UNBALANCED) JOINT   CONSTANT·STIFFNESS (VARIABLE-THICKNESS) JOINT

ADHESIVE SHEAR STRESS DISTRIBUTION   ADHESIVE SHEAR STRESS DISTRIBUTION
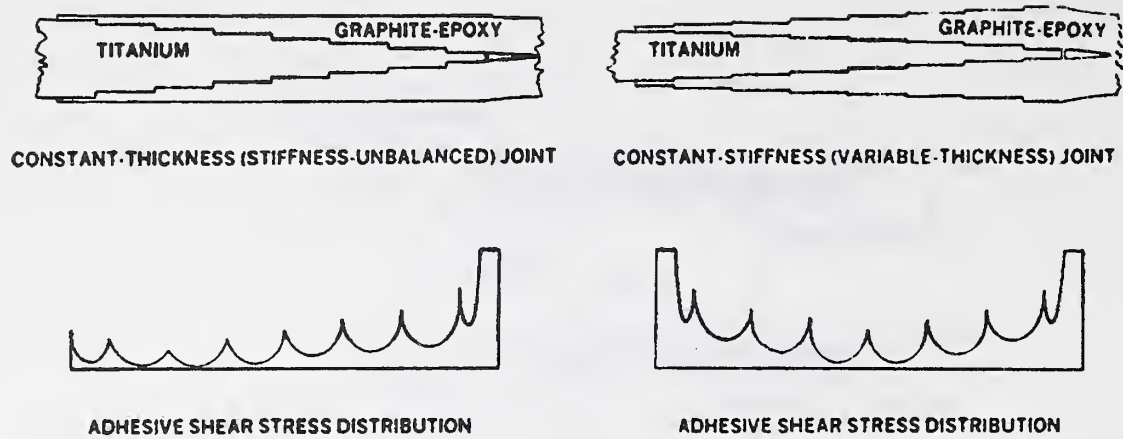
Figure 3.14 Stepped lap bonded joints (strength improvement by matching stiffnesses)
(from Hart-Smith, 1987)
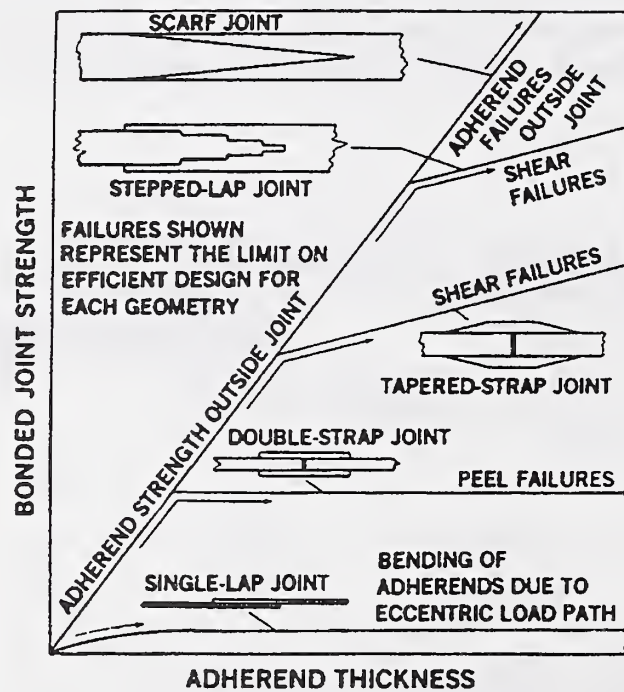
Figure 3.15 Relative strength of different bonded
joint types (from Hart-Smith, 1987)

Single lap

Single lap

Toggle lap

Double lap

Beveled double lap

Double lap

Double lap

Step lap

Step lap

Multiple step lap

Tongue and groove

Simple scarf

Simple scarf

Double scarf

Double scarf

Corner single lap

Corner single lap

Corner double lap

Intersecting wall lap
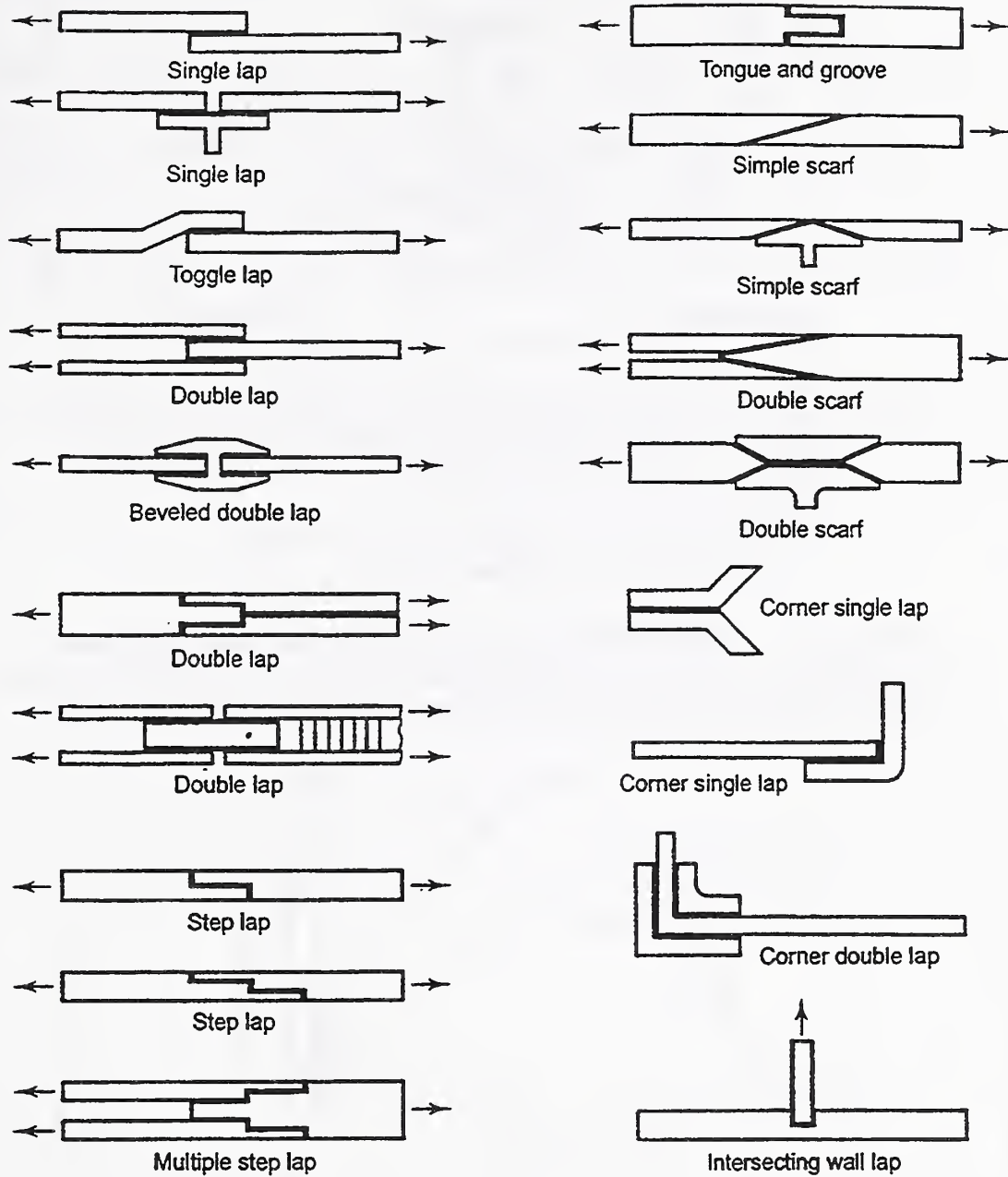
Figure 3.16   Bonded shear joint concepts
(from Hutchinson, 1997).
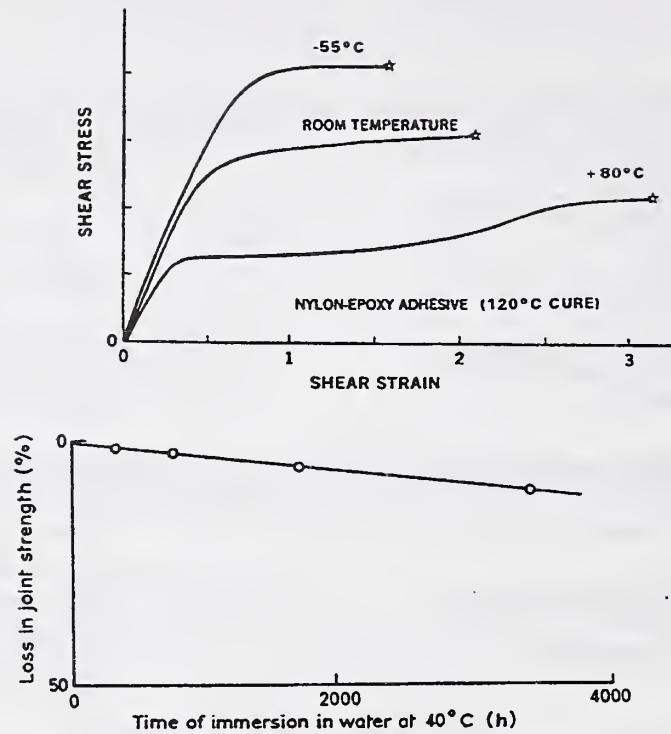
Figure 3.17 (Upper) Effect of temperature on adhesive shear behavior (from Matthews, 1987). (Lower) Effect of immersion in water of FRP / two-part acrylic adhesive / joints on tensile lap shear strength (from Kinloch, 1987).
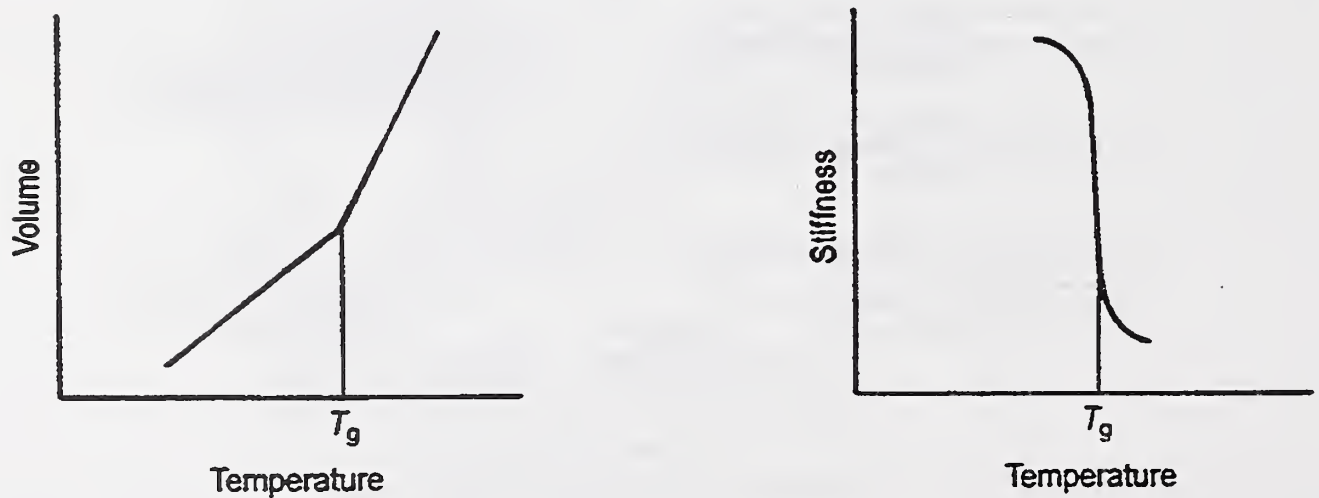
Figure 3.18 The glass transition temperature (from Hutchinson, 1997).
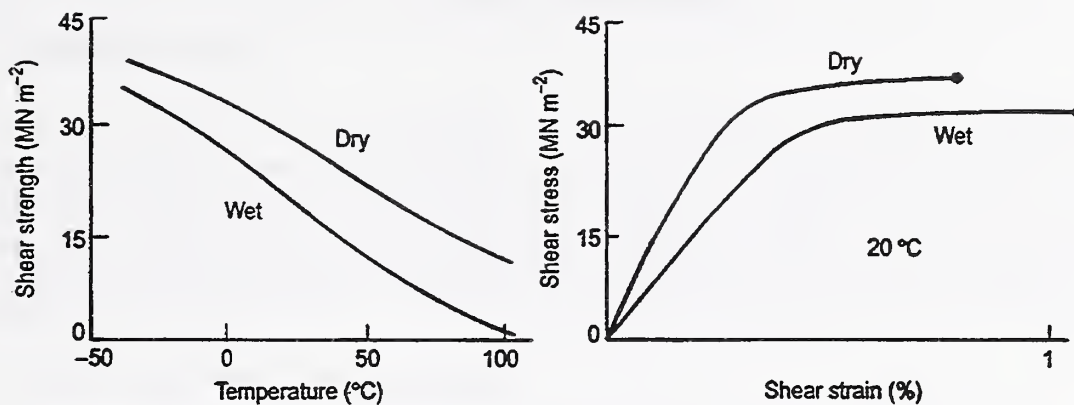
Figure 3.19   Typical effects of temperature and moisture on the mechanical
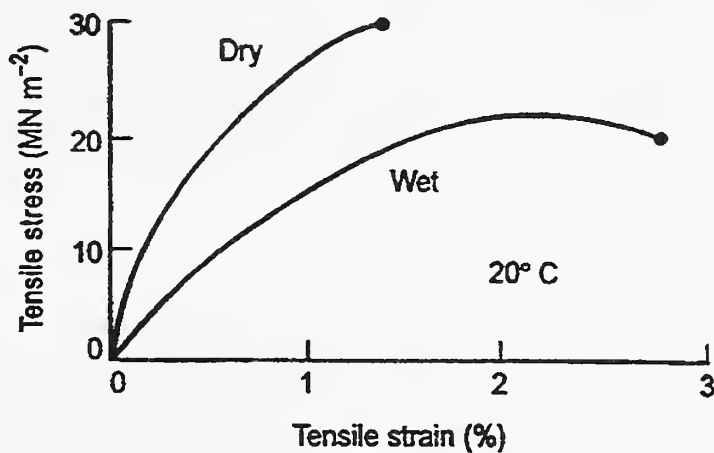behavior of an unmodified cold-curing epoxy adhesive in shear.
(from Hutchinson, 1997)

Figure 3.20   Typical effects of temperature and moisture on the
mechanical behavior of an unmodified cold-curing epoxy
adhesive in tension (from Hutchinson, 1997)

Chapter 4
Frame Connections

4.1 INTRODUCTION

The previous chapters covered connections designed primarily to transmit in-line forces, with secondary bending moments caused by the geometry of the connectors. These types of connections have benefited from research in aerospace engineering. In contrast, frame connections (between beams and columns) are primarily of concern to civil engineers and are designed to transfer moments as well as shear and in-line forces between thick members, such as I-sections.

4.2 BEAM-COLUMN CONNECTIONS

Beams transfer floor and roof loads to columns to which they are joined by rigid (moment-resisting), flexible (free to rotate), or semi-rigid connections. For uniformly loaded beams, flexible connections (or simple supports) cause the maximum moment to occur at midspan, whereas rigid connections (or fixed supports) shift the maximum moments to the ends. Semi-rigid connections provide an opportunity to equalize the maximum negative moments (upward bend) at the supports with the maximum positive moment (downward bend) at midspan, an efficient design for a symmetrical cross-section, such as an I-shape (Fig. 4.1).

In steel buildings, beam-column connections usually consist of angles or cleats bolted or welded to the flanges of the column and the beam, or the flanges of the column and the web of the beam. In addition, gusset plates or stiffeners are sometimes used where necessary to tie the column flanges together, thus transferring the beam load to both column flanges and mitigating web crippling. The same designs are essentially carried over to FRP beam-column connections, which include bolts only or a combination of adhesive bonding and bolts. Whereas steel frame connections are designed primarily for (ultimate) strength limit state (failure under overload conditions), concern with FRP connections is more with limiting deflections (serviceability limit state) and preventing instability.

4.3 PINNED CONNECTIONS

Figure 4.2 shows a FRP connection consisting of an angle bolted to the web of an I-section and designed to act essentially as a pinned connection. It must be able to resist shear force and accommodate rotation. The use of adhesive alone is not recommended because it may lead to catastrophic failure at small rotation or deflection due to inter-ply splitting of the web angle. Bolting alone can satisfy both ultimate and serviceability limit states, e.g., the maximum deflection must be less than 1/250 of the beam span [EUROCOMP (Clark, 1996)]. A combination of bonding and bolting enables the connection to incur no slip even at the serviceability limit, but localized cleat failure may occur.

The EUROCOMP Design Code (Clark, 1996), which is a practical design code limited to glass FRP components, connections and assemblies for the construction industry, recommends bolt hole clearance of 2 mm (0.08 in) for constructibility, and a gap of 10 mm (0.4 in) between beam

end and column faces to allow rotation. The EXTREN Design Manual (1989), published by Strongwell Corporation for the design of glass FRP structural shapes, assumes the use of FRP bolts at least 10 mm (0.4 in) in diameter, in holes with clearance of 1 mm to 2 mm (0.04 in to 0.08 in), and tightened up to a torque of 24 N-m (210 lbf-in).

4.4 SEMI-RIGID CONNECTIONS

Figure 4.3 shows a typical failure of a bolted FRP beam-column connection, in which angle sections are used to join the flanges of a beam and a column. The FRP angle fails by formation of hinges due to opening (top) or closing (bottom) moments. Other likely locations of failure are under the bolts going through the column (crushing) and at the junction of web and flange of FRP I-beams, an area relatively rich in resin and poor in fibers (Fig. 4.4). The simplest solution is to replace the FRP angles with stainless steel ones, if weight is not a concern.

Mottram and Zheng (1997) improved on the basic design of semi-rigid connections by:
- Increasing the stiffness of the connecting angles by doubling their thickness or adding more angles (Fig. 4.5).
- Stiffening the flanges of beam and column with bolts (Fig. 4.6).

These FRP semi-rigid connections are stiffer, stronger and rotate further before failure than steel connections (Fig. 4.7). The combination of bolting and bonding improves performance over bolting or bonding alone (Fig. 4.8).

Mosallam (1994) increased the stiffness of the connecting angle by adding triangular gusset plates. This so-called "universal connector" is especially made with molded glass / vinyl ester but is not yet a readily available item (Fig. 4.9). The use of over-stiff connectors transfers failure to the FRP beam and column, so they must be stiffened as well (Figs. 4.10 and 4.11). Alternatively threaded rods may also be used in conjunction with the "universal connector". One criticism of the tests performed by Banks, Mosallam and McCoy (1994) is that their test set-up produces both bending and compression in the members (Fig. 4.12). This compression is usually not present in the beam of actual frames and may explain some of the failure modes observed.

4.5 REFERENCES

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Uniformly distributed loading
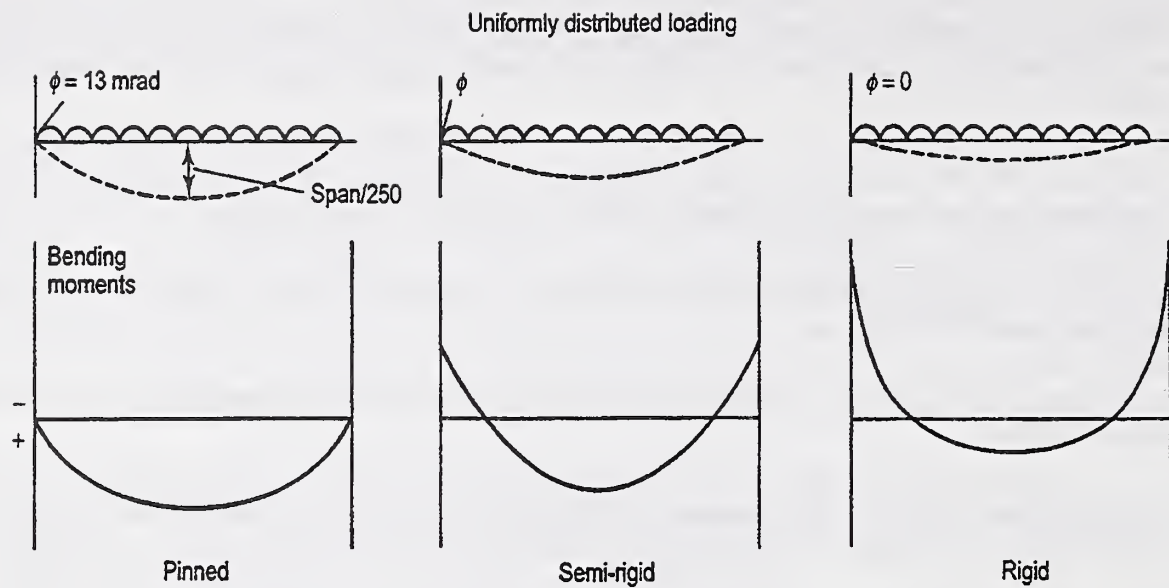
Figure 4.1 Influence of end-restraint on bending moment distribution
and deflection of a beam (from Mottram and Zheng, 1996)
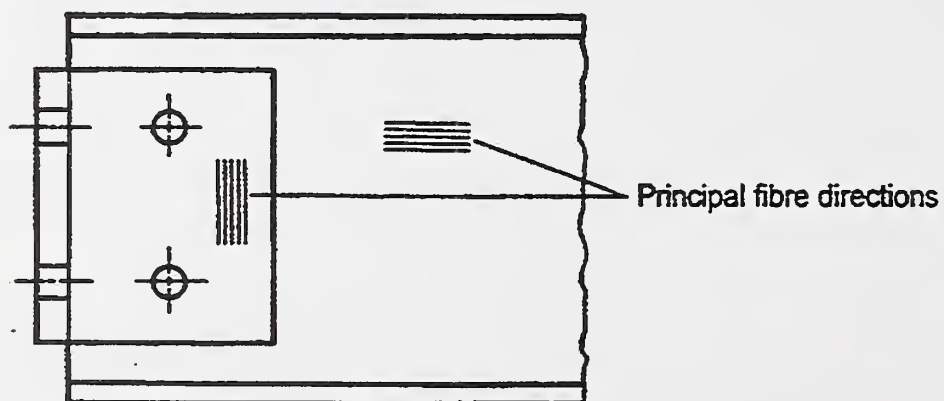
Figure 4.2 Web-cleated connection
(from Huchinson, 1997)

Composite failure
in L-shaped cleat
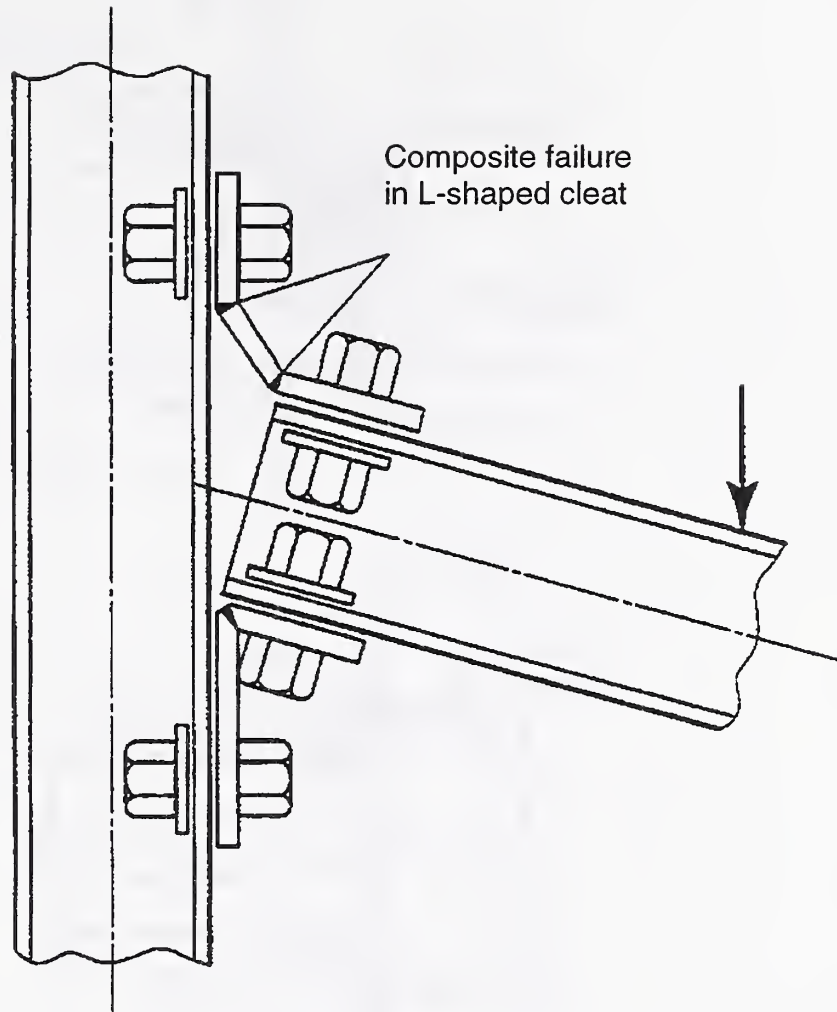
Figure 4.3 Failure of composite
cleat.   (from Hutchinson, 1997)
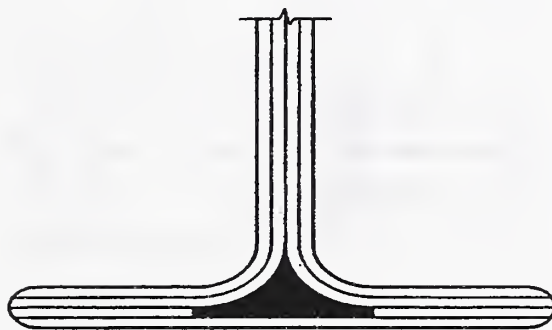
Figure 4.4 Resin-rich zone at flange/web junction of
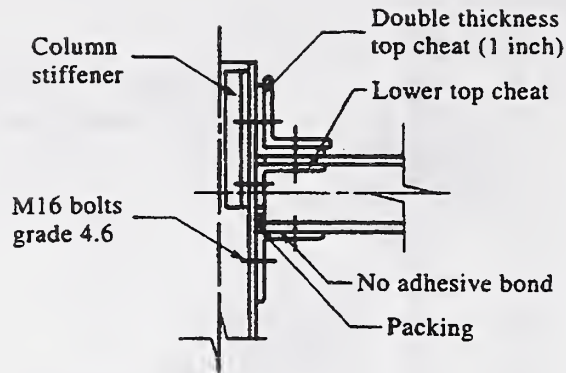pultrusion (from Hutchinson, 1997)
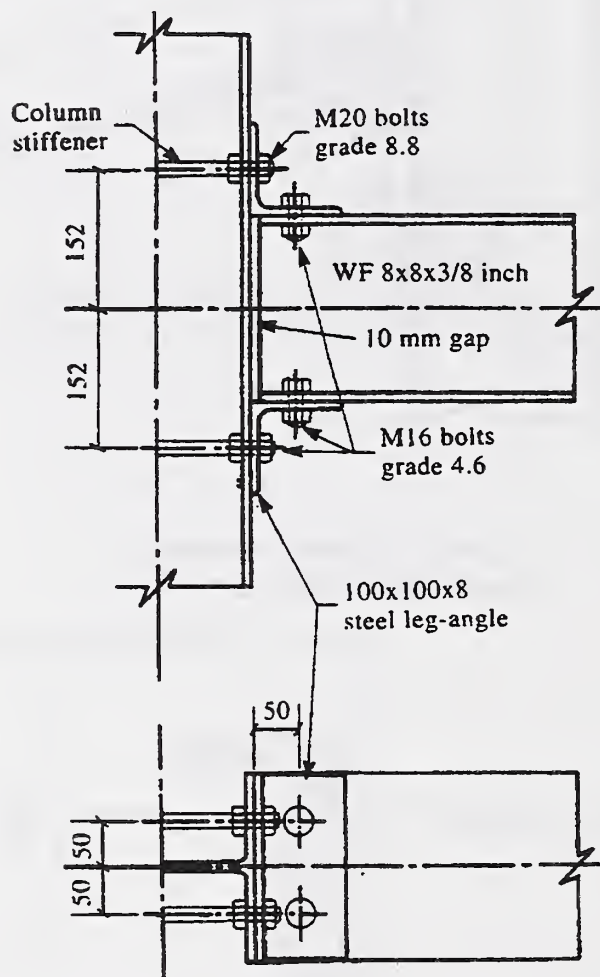
Figure 4.5 Detail of FRP connection DTLmj (top) and steel connection Stmj (bottom two). (from Mottram and Zheng, 1996)
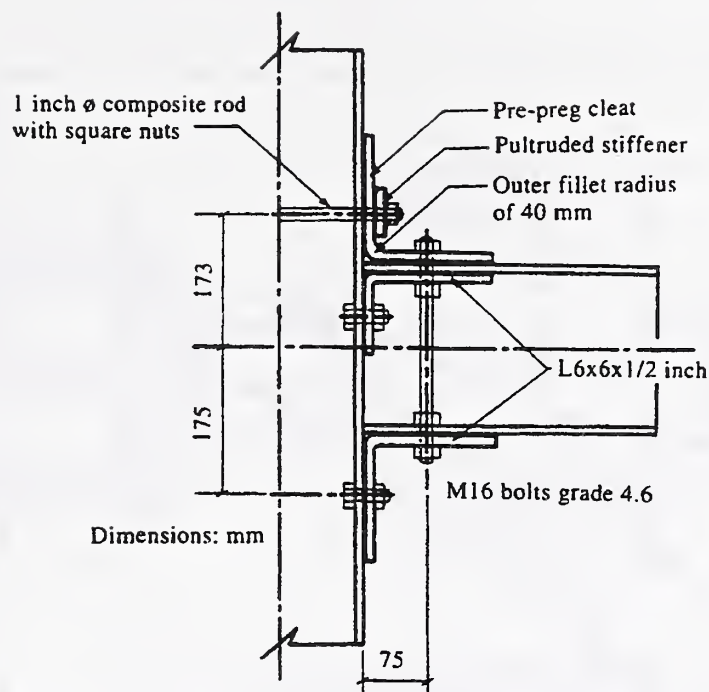
Figure 4.6 FRP connection TLmj
(from Mottram and Zheng, 1996)
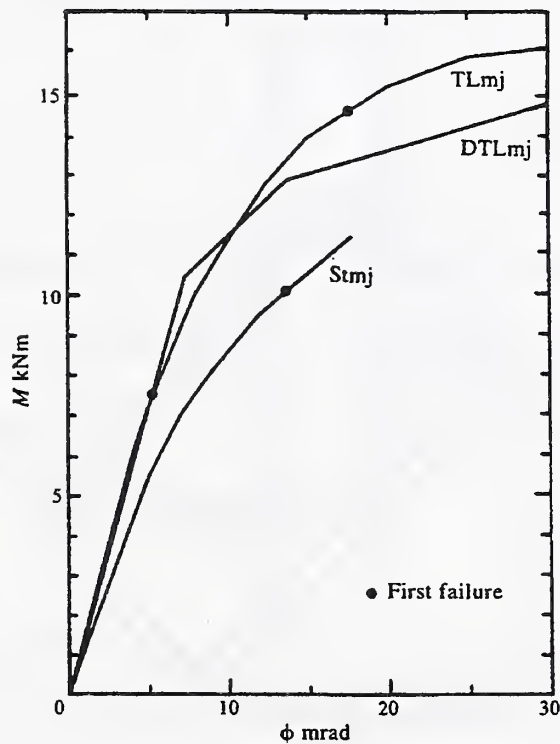
Figure 4.7 Moment-rotation curves of isolated semi-rigid connections
DTLmj = Double thickness top cleat, single thickness lower cleat, bending about major axis;
TLmj = top and lower cleats, major axis; Stmj = Steel connection of steel member, major axis.
(from Mottram and Zheng, 1996)
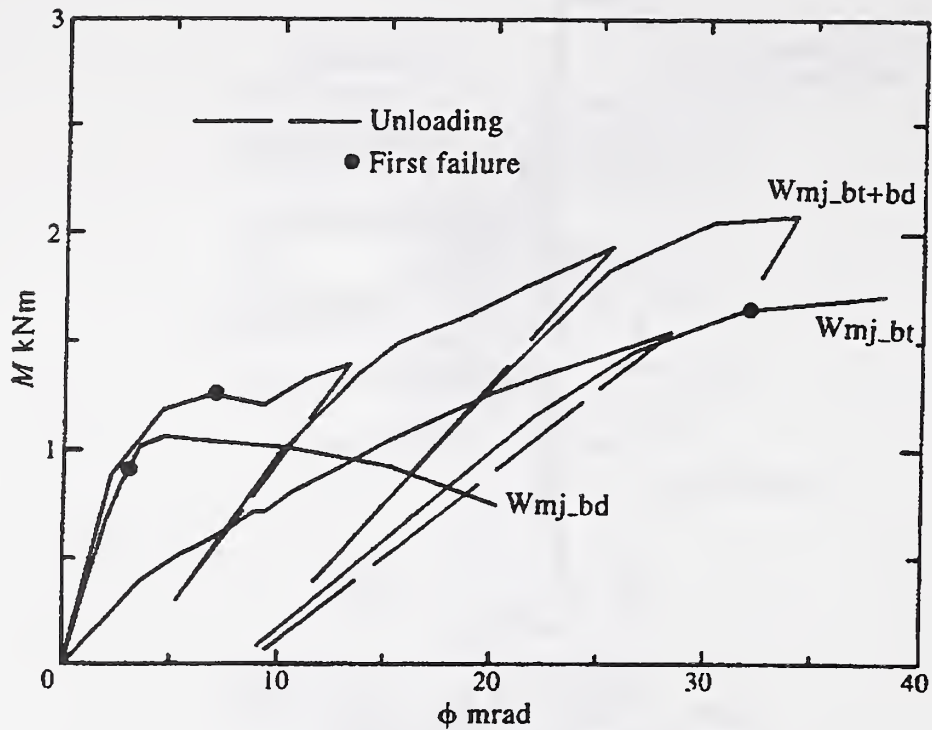
Figure 4.8 Moment-rotation curves of isolated pinned connections
(Wmj = web cleat, bending about major axis, bt = bolting, bd = bonding)
(from Mottram and Zheng, 1996)
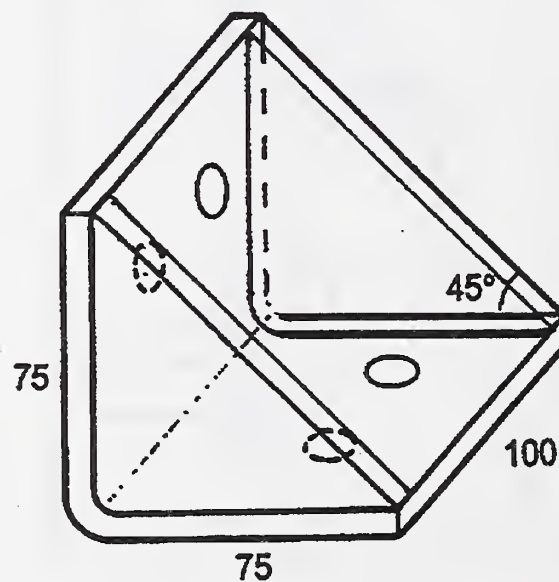
Figure 4.9 Molded FRP "universal
connector" (from Mosallam, 1994)
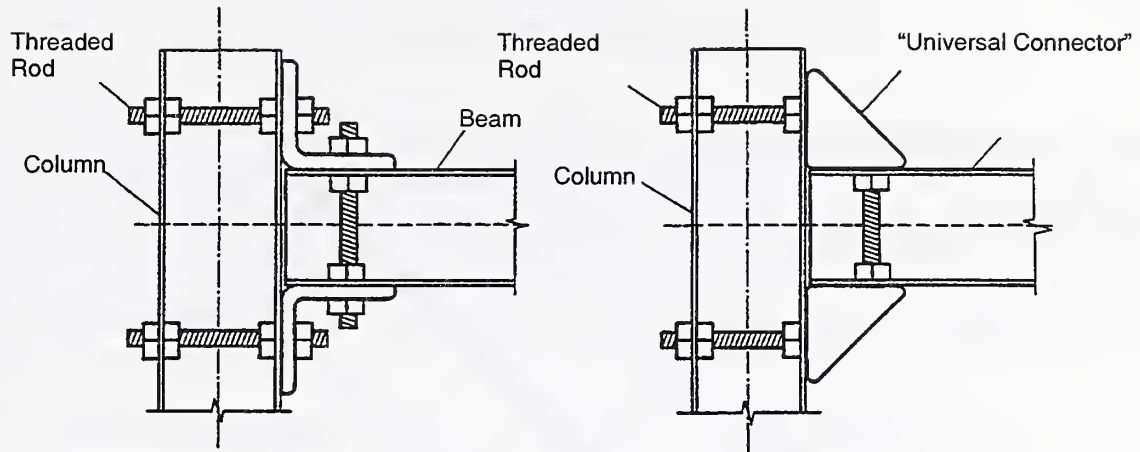
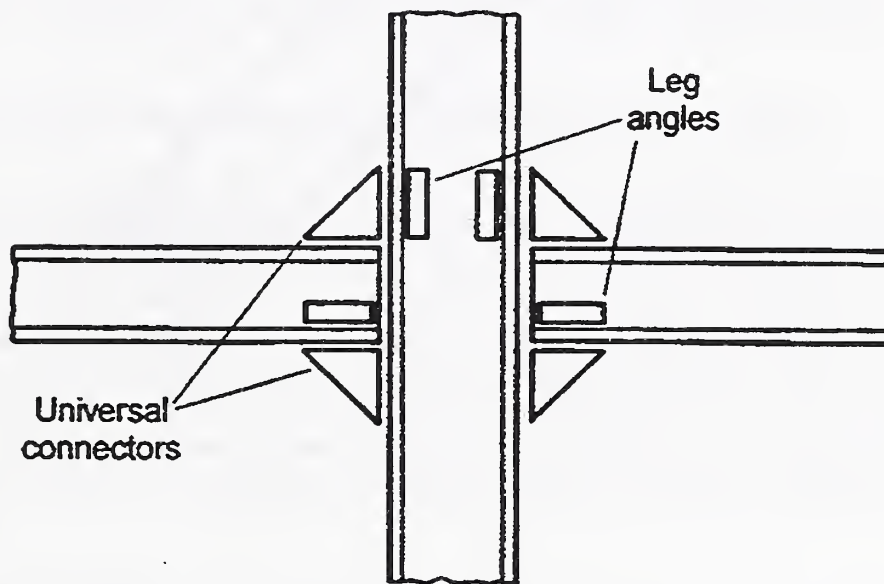Figure 4.10   Use of threaded rod stiffeners
(from Mosallam, 1994)

Figure 4.11   Thickening and stiffening of flanges of column
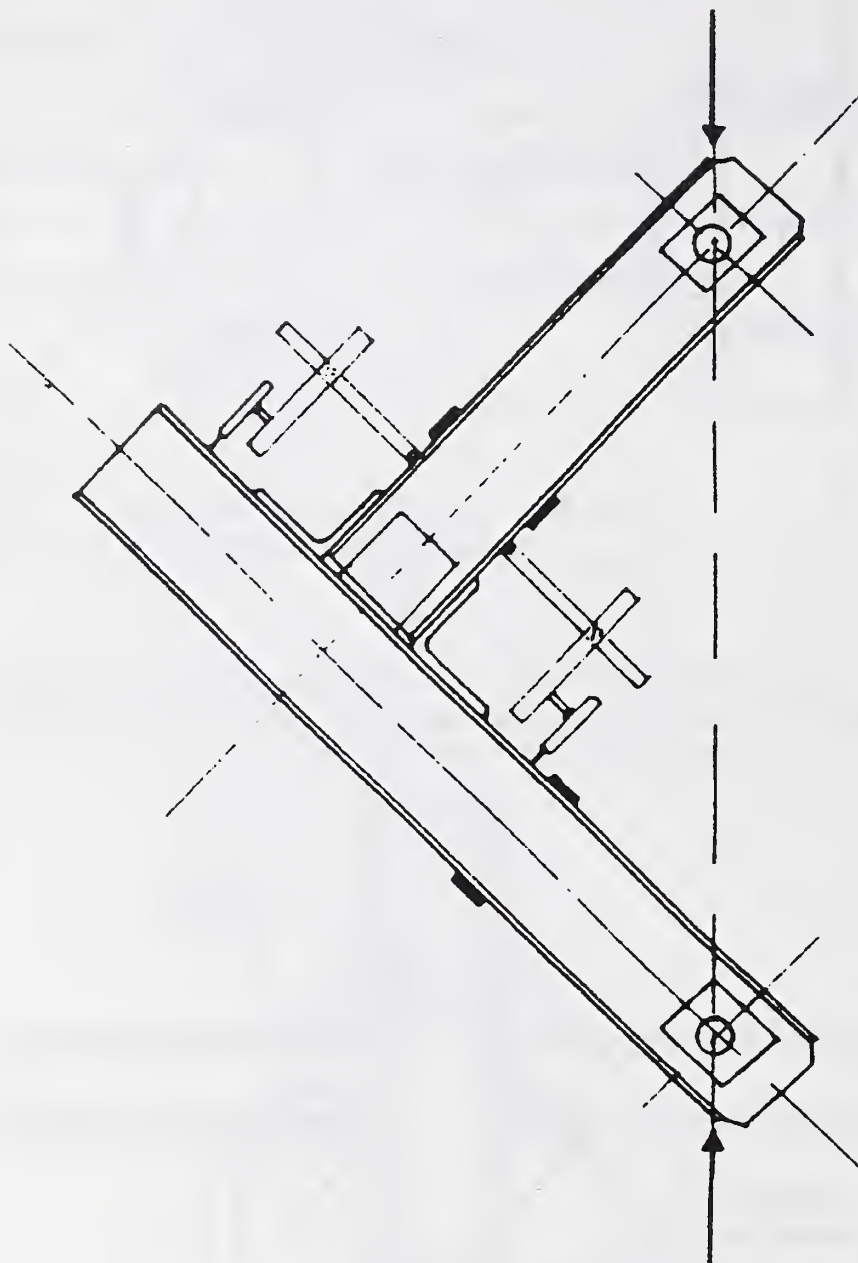and beam using leg angles (from Mosallam, 1994)

Figure 4.12   Schematic of connection test setup.
(from Banks et al., 1994)

Chapter 5
Interlocking Connections

A major advantage of FRP construction is the ease of transportation and construction that comes with light-weight members. Interlocking joints, sometimes bolted or bonded as well, are particularly well suited for FRP construction because they allow connections to be made quickly. Geometric interference and surface friction may provide sufficient integrity, without any adhesive or mechanical fasteners, for some large, interlocking structures to carry light loads. The main disadvantage of such connections is that members must be fabricated to close dimensional tolerances.

5.1 TUBES AND PANELS

Tubular FRP members benefit from a long tradition of pipe fitting. Figures 5.1 and 5.2 show some examples of interlocking and bonded joints for circular and square tubing. Figure 5.3 shows a system of construction of sandwich panels that can be assembled quickly by inserting them into ready-made corner and other joints, followed by bonding or bolting. The panels carry flexural loads and consist of glass FRP skins bonded to a low-density core.

The interlocking connectors should maximize flexibility of orientation of standard components. A truly universal connector requires a large number of fibers to run in as many directions as possible, which is difficult to achieve with pultrusion. Rather, a molding process is used for special or low volume connectors.

5.2 ADVANCED COMPOSITE CONSTRUCTION SYSTEM (ACCS)

Interlocking joints are mostly ingenious, proprietary inventions protected by patents. The ACCS consists of pultruded panels joined together by special pultruded dog-bone connectors that can also be adhesively bonded, if dismantling of the cellular, monocoque structure is not planned (Fig. 5.4). (A monocoque structure possesses sufficient strength and stiffness by itself, here through mechanical interlocking alone, and does not require additional framing). The connectors are constructed with ridges that control the thickness of the bond line (Fig. 5.5). The system is designed for stresses up to 5.0 kPa (0.7 psi) and a serviceability limit strain of 0.3 % (Hutchinson, 1997). It is suited for one or two story field offices or emergency buildings (Fig. 5.6) and bridge enclosures (designed to protect structural members permanently from the environment).

5.3 "SNAP" JOINTS

The "SNAP" joint is an interesting type of connection that locks in place and cannot be dismantled. Figures 5.7 and 5.8 show pultruded members whose fiber architecture is designed so that the load-bearing faces of the ridged connectors have very high interlaminar shear strength. These connections were an integral part of a 28 m transmission tower (Fig. 5.9) that was assembled on its side, then uprighted by a crew of three in a few hours (Goldsworthy and Hiel, 1998). Light weight was crucial to this construction and allowed significant cost savings. The

system has been expanded further to include members that would be found in a multi-story building (Figs. 5.10 and 5.11).

5.4   CONCLUSION

Interlocking connections use the light weight and ease of FRP to be fabricated into custom-made shape to advantage. Although they are adaptations of historical designs with timber and cast iron, they represent a significant and promising departure from steel-like connections.

5.5   REFERENCES

Goldsworthy, W.B. and Hiel, C. (1998), "Composite Structures are a SNAP," *Proc. Second International Conference on Composites in Construction*, ICCI 98, Tucson, Arizona., USA, **2**, pp. 382-396, Jan.

Head, P.R. and Churchman, Q.E. (1989), "Design, Specification and Manufacture of a Pultruded Composite Construction System." *Proc., BPF Symposium on Mass Production Composites*, Imperial College, London, Sep., pp. 117-162.

Hutchinson, A.R. (1997), "Joining of Fibre-reinforced Polymer Composite Materials," *CIRIA Report 46*, Nov.
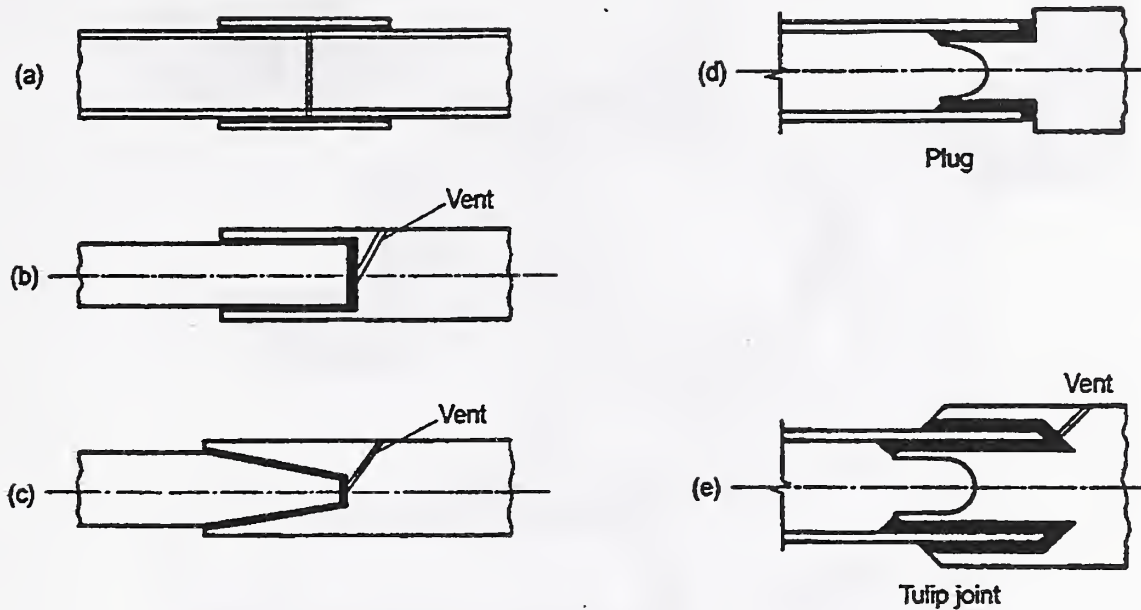
Figure 5.1 Examples of bonded tubular joints
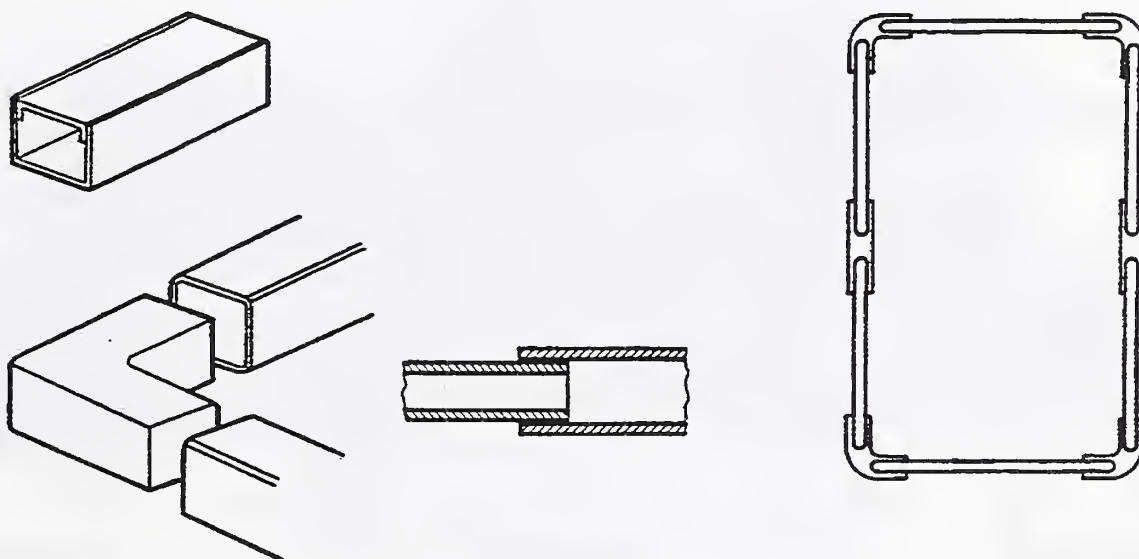(from Hutchinson, 1997)

Figure 5.2 Examples of bonded interlocking system components
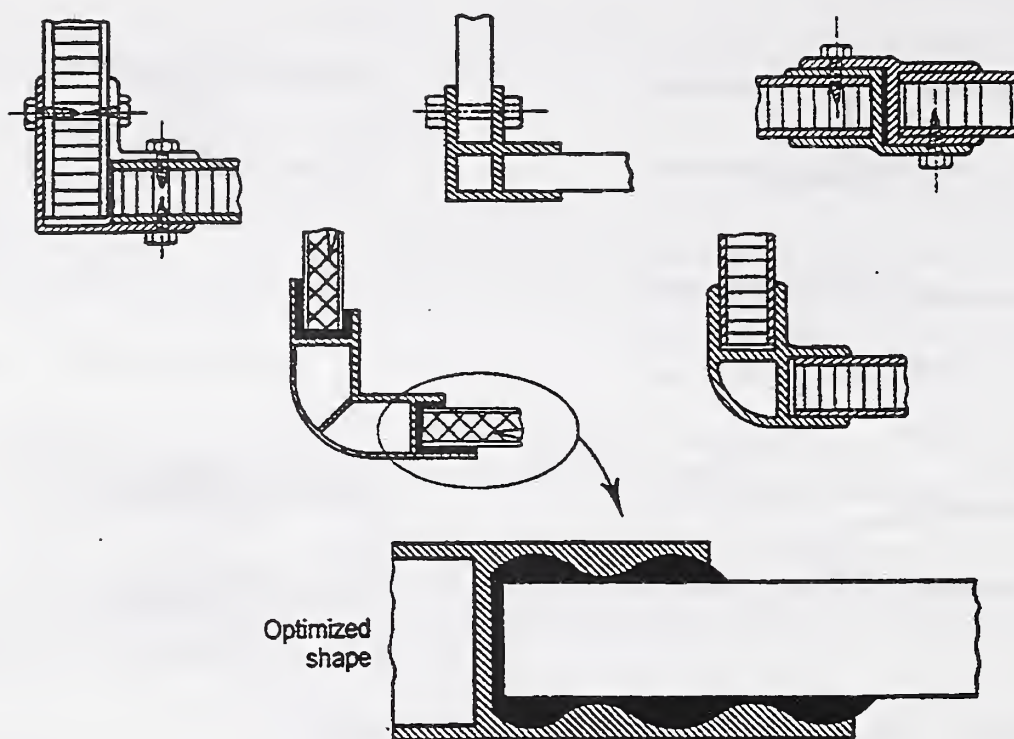(from Hutchinson, 1997)

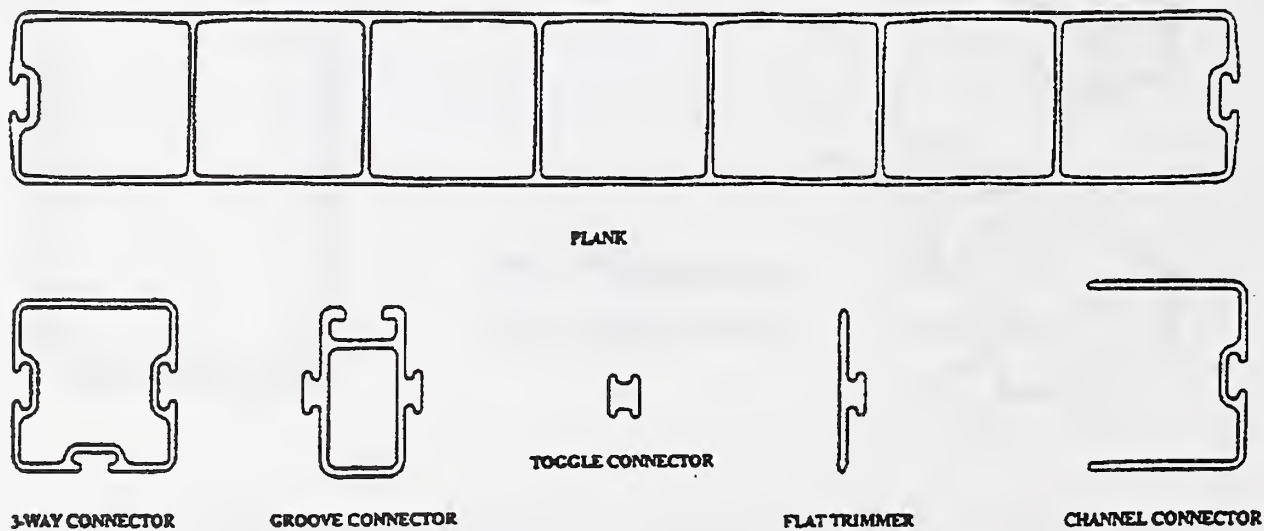Figure 5.3 Concepts for sandwich panel connections
(from Hutchinson, 1997)

PLANK

3-WAY CONNECTOR   GROOVE CONNECTOR   TOGGLE CONNECTOR   FLAT TRIMMER   CHANNEL CONNECTOR

Figure 5.4 Principal components of advanced composite
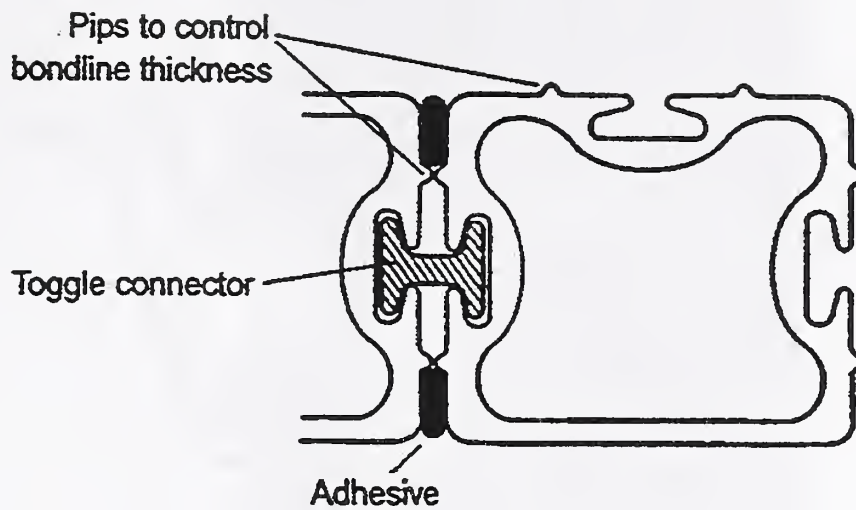construction system (ACCS) (from Head and Churchman; 1989)

Figure 5.5 Pips or ridges molded into surface of composite to control bondline thickness (from Head and Churchman, 1989)
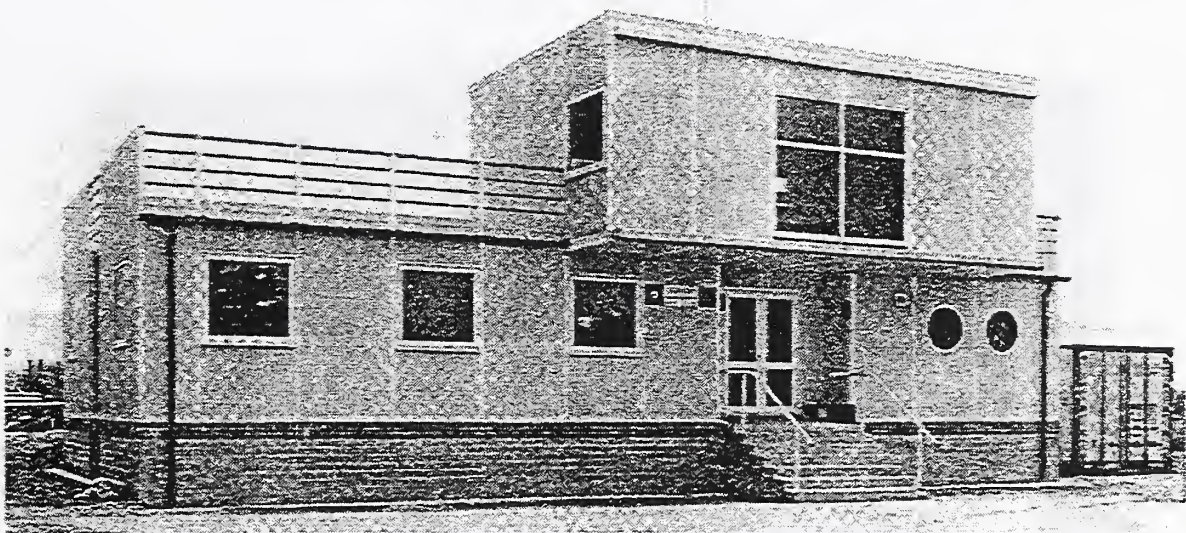
Figure 5.6 Advanced composite site office building, Second Severn Crossing, Avon, UK. (from Head and Churchman, 1989)
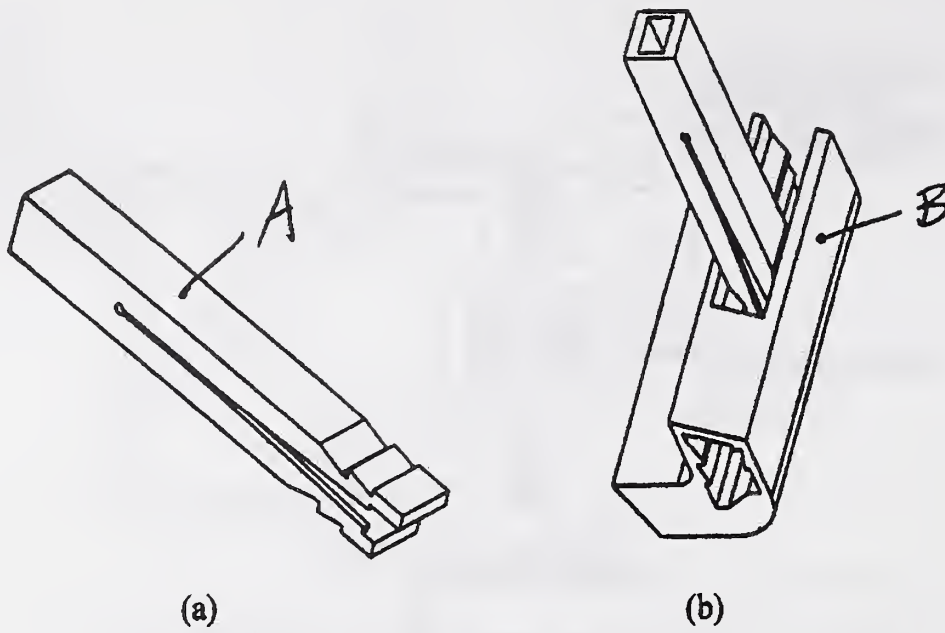
(a)   (b)

Figure 5.7 (a,b) "Snap" joint concept
(Designed by W.B. Goldsworthy & Associates, Inc.)
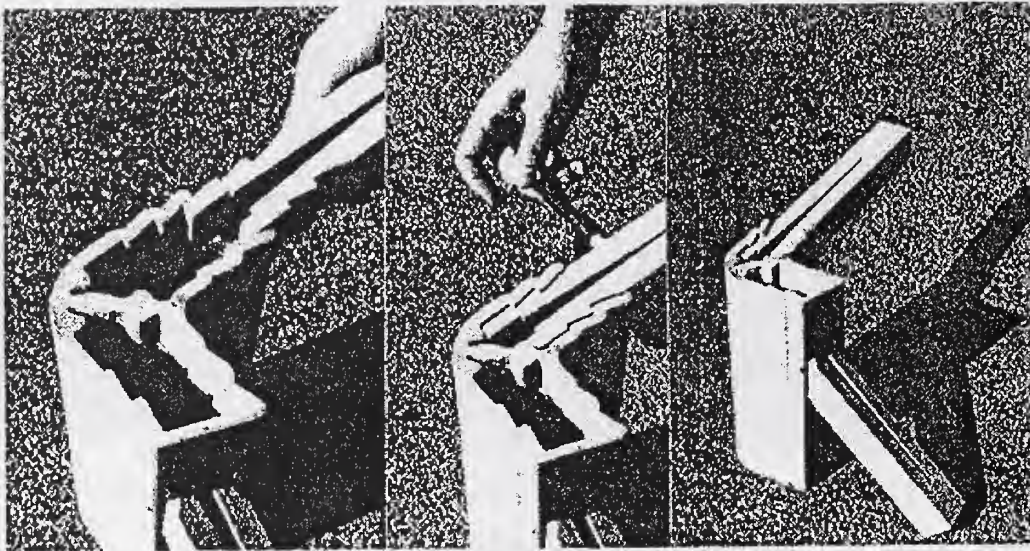(from Goldsworthy and Hiel, 1998)

Figure 8 (a, b, c) Hardware for fasteners "Snap" joint
(Designed by W.B. Goldsworthy & Associates, Inc.)
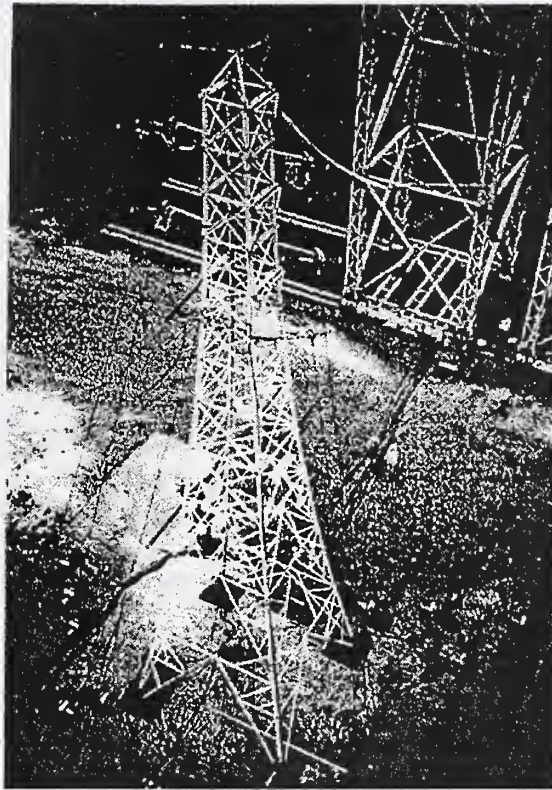(from Goldsworthy and Hiel, 1998)

Figure 5.9 Composite transmission tower with SNAP joints
(Designers: W.B. Goldsworthy & Associates, Inc. 1994 for Ebert Composites)
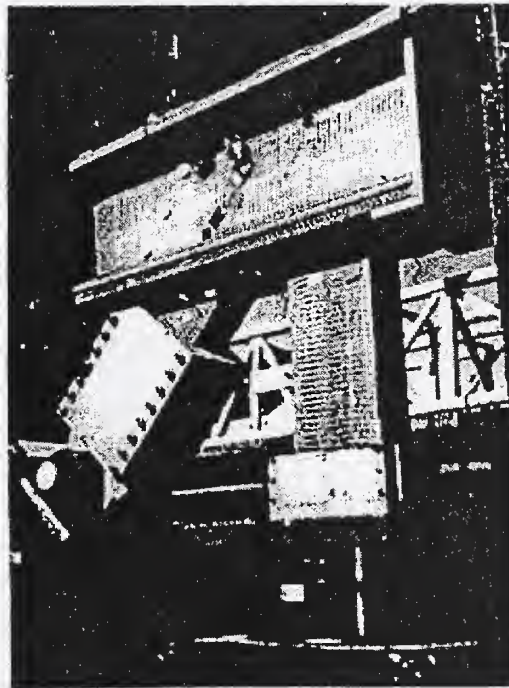(from Goldsworthy and Hiel, 1998)

Figure 5.10 Test setup for truss joint
(at Coordinated Equipment Index Testing Facility, Wilmington, CA)
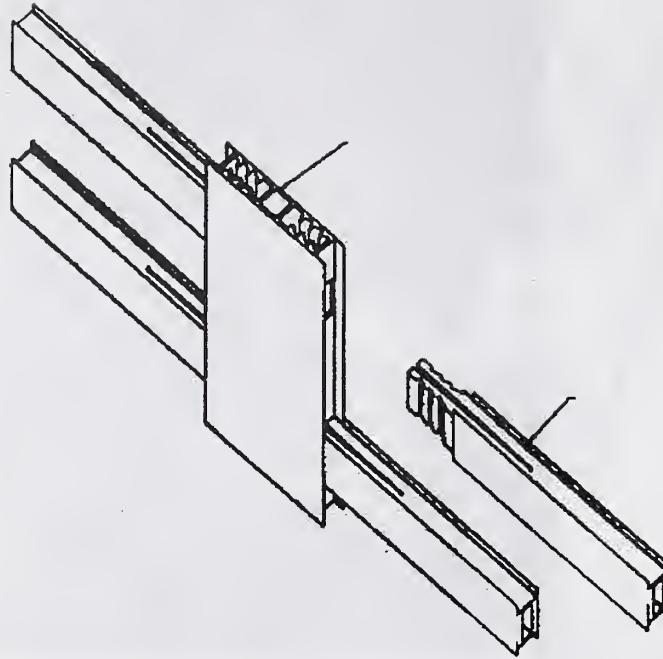(from Goldsworthy and Hiel; 1998)

Figure 5.11 "Snap" joint design for composite high rise
(Designed by WBG&AAI for Commercial Spaceport .S.A.)
(from Goldsworthy and Hiel, 1998)